PRACTICAL PHYSICS

BARTON AND BLACK

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AN INTRODUCTION TO
PRACTICAL PHYSICS
FOR
COLLEGES AND SCHOOLS

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SECOND EDITION

THIRD IMPRESSION

LONDON
EDWARD ARNOLD & CO.
1922

PREFACE TO FIRST EDITION

This little work is intended for students beginning Practical Physics, whether in the upper forms of schools, or at University or Technical Colleges. Although not assuming any previous conversance with the subject, it provides more than sufficient for the Intermediate Examination for degrees in Pure Science or Engineering, and will be found suitable also for University Scholarship Candidates.

It comprises all the various branches of elementary physics, and has been written with a view to the use of apparatus which is either of standard type or of simple forms easily provided in any laboratory.

Experience has shown that some points are frequent stumbling-blocks to certain classes of students, and not less so when they are unduly laboured by *printed* instructions. These are accordingly indicated in the text as possibly needing *oral* elucidation by the demonstrator. It is thus hoped that the letterpress is thereby reduced to a minimum without any loss of essential clearness.

At the head of each experiment stands a list of apparatus needed. This facilitates its setting out by the assistant, and serves to show the student whether he has what he needs. Although the book is divided into seven parts, each experiment is identified by its number simply, since these numbers run consecutively through the book.

Each experiment contains an explicit statement of the exercises to be performed by the student and of the form in which his results should be presented.

The selection of the 120 experiments has been made with

the intention of providing a sufficient central course or nucleus for one or two years' work. In every laboratory new experiments are continually being introduced and tried. And, with such extensions, many students have proceeded to the B.Sc. degree without possessing any more advanced work than the present introductory volume.

Tables of logarithms and trigonometrical ratios are provided at the end. Any other physical constants needed are presumed to be accessible in the laboratory's books of reference.

Acknowledgments are hereby gratefully tendered to the Controller of H.M. Stationery Office for permission to reprint from "Mathematical Tables for the Use of Students" issued by the Board of Education, the tables of logarithms and of trigonometrical ratios on pages 182–184, and to Messrs. Macmillan & Co., Ltd., for their permission with respect to the logarithms of numbers from 1000 to 2000 on the upper part of page 182.

E. H. B.

T. P. B.

NOTTINGHAM,
June, 1912.

PREFACE TO SECOND EDITION

THE text has been revised and now embodies the alterations that experience has shown to be desirable.

Users of the book will regret to learn that, shortly after assisting in the preparation of this new edition, Captain Black went out to the East and there fell in action.

E. H. B.

CONTENTS

PART I

GENERAL PHYSICS

PART II

HEAT

PART III

LIGHT

PART IV

SOUND

PART V

MAGNETISM

PART VI

ELECTROSTATICS

PART VII

ELECTRIC CURRENTS

AN INTRODUCTION TO
PRACTICAL PHYSICS

PART I.—GENERAL PHYSICS

1. Verniers.

APPARATUS.—Models of Common Verniers, Calliper Gauge, Model of Barometer, Scales and Verniers, Sextant, Spectrometer, Barometer, Objects of which lengths and thicknesses may be measured.

OBJECT.—To read verniers, which are devices to give subdivisions of a scale division.

THEORY.—The vernier consists of a small scale, which runs along the scale proper. The vernier scale is so made, that a certain number of its divisions are equal to a certain number of the scale divisions. For instance, in the simplest arrangement, 10 vernier divisions = 9 scale divisions, or, in symbols $10V = 9S$. This particular combination is one of a whole class of scale-vernier combinations, in which the relation between a scale division and a vernier division is given by $nV = (n - 1)S$, where n is an integer.

Taking this class first, consider the arrangement mentioned above, where $10V = 9S$. Obviously in this case a vernier division is less than a scale division by $\frac{1}{10}$ of a scale division, and, in general, a vernier division is less than a scale division by $1/n$ of a scale division. So that, if the scale is in mms., the difference between a scale division and a vernier division

is 0·1 mm. and the vernier enables one to read accurately to 0·1 mm.

METHOD.—How to use the vernier will be clear from Fig. 1.

Let SS be the scale, VV the vernier, and AB a rod of which the length is required. The required length is 3 mms. and a fraction. To obtain this fraction look along the vernier, until you find a vernier division which is exactly opposite a scale division. In this case it is the fourth. Therefore since CD contains 4 scale divisions and EF contains 4 vernier divisions and the difference between a scale division and a vernier division is 0·1 mm., the required fraction is 0·4 mms.

The rule to be applied is, therefore, as follows :—look along the vernier until you find a division which is opposite a scale division. Let this division be m. Then, in the case

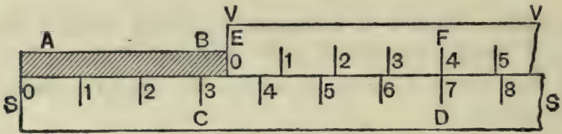

FIG. 1.—Forward-reading vernier.

above, the required fraction is $m/10$ of a scale division and, in general, m/n of a scale division.

EXERCISE.—(1) Read the lengths and thicknesses of the given objects by means of the calliper gauge.

(2) Examine the vernier on the inches side of the model of the barometer scales supplied. Decide how it reads. Take some readings.

(3) Examine the vernier on the sextant. Take some readings of angles.

(4) Examine the verniers on the spectrometer. Read off some angles.

Ask a demonstrator to check some readings taken by you with each of the above.

A second type of vernier is one in which $nV = (n + 1)S$.

So that, in general, a vernier division is $1/n$ of a scale division greater than a scale division.

From Fig. 2 it will be obvious that the length AB is again 3·4 mms. In this case a vernier division is $\frac{1}{10}$ greater than a scale division, and since the required coincidence takes place at the sixth vernier division, the difference between the

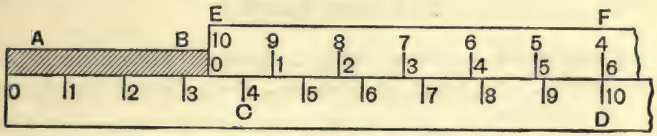

FIG. 2.—Backward-reading vernier.

lengths EF and CD is $\frac{6}{10}$ of a scale division, and therefore the required fraction is $\frac{4}{10}$ of a scale division (*i.e.* $1 - \frac{6}{10}$).

In order to avoid the subtraction, the vernier scale is generally numbered backwards as in the top row of figures on it in the figure. So that the reading may be taken exactly as in the first type of vernier.

The advantage of this type over the first is that the vernier divisions are larger and the coincidence is more easily seen.

EXERCISE.—Take some readings with the vernier, which is provided, of this type.

A third type of vernier is one in which $nV = (2n - 1)S$. This vernier again reads, in general, to $1/n$ of a scale division. To see how this is, consider the case $10V = 19S$. If the vernier divisions were each divided into 2, then $20V$ would be equal to $19S$, and in that case the vernier would read to $\frac{1}{20}$ of a scale division. But since the vernier divisions are not divided into 2, it can only read to $\frac{2}{20}$, $\frac{4}{20}$, &c., *i.e.* to $\frac{1}{10}$, $\frac{2}{10}$, &c., of a scale division.

In this type the vernier divisions are very large, and the coincidence is easily seen.

EXERCISE.—Examine the vernier on the cms. side of the model of the barometer scales and take some readings. Finally go to the barometer and practise reading its verniers.

Then ask a demonstrator to check some readings which you make.

Results.—Write a brief outline of all you have observed, making sketches of the various types of verniers and scales dealt with.

2. Screw Gauges.

Apparatus. — Screw Gauge, Electric Micrometer, Leclanché Cell, Telephone, Spherometer, Plane Glass Plate, Wires and Metal Discs.

Preliminary.—By means of the instruments mentioned in this experiment, greater accuracy in measuring small lengths is obtained than by the use of a vernier. This is accomplished by means of the motion of a screw. If the screw has a large head with a scale marked round the head, a minute fraction of a complete turn of the screw can be read off on this scale and thus a very small forward motion of the point of the screw can be determined.

1. Screw Gauge.—Before using the instrument note carefully what the pitch of the screw is in terms of the linear scale marked on the side of the gauge, and also to what fraction of a unit of the linear scale each division of the circular scale corresponds. Then you should have no difficulty in using the instrument.

Zero Error.—Very often the zero of the linear scale does not correspond exactly to the "closed" position of the instrument. There is then what is called a "zero error." The instrument must always be examined at the outset for this error, and if there is such an error, it must be determined and taken into consideration in all measurements.

Exercise.—Find as accurately as possible the thickness of the wires and plates provided.

Note.—Do not screw the screw too hard on to the object of which the dimensions are required. It is difficult to tell exactly when the screw is just gripping the object. Care must be taken regarding this. In the *electric micrometer*, mentioned

below, a means is provided of telling when the screw is just touching the object, provided the object is of a material which conducts electricity.

2. Electric Micrometer.*—The principle is the same as in the screw gauge. In general, the head of the screw is large and the pitch of the screw small, so that it is often possible to read easily to 1/1000 mm.—a length, which is called a *micron* and denoted by μ.

When the point of the screw touches the object, the contact completes an electric circuit, in which a telephone is inserted, and the fact, that the screw is touching the object, is made evident by a sound in the telephone.

Method.—Find the value of the divisions of the linear and circular scales on the micrometer. Connect up as shown in Fig. 3.

C is a piece of plate glass. Turn the screw until the point is just touching the conducting base B, as indicated by a sound in the telephone. Take the readings of the linear and circular scales. Screw back the screw. Insert the object A, of which the thickness is required. Screw down the screw until the point is just touching A. Read the scales and thus determine the thickness of A.

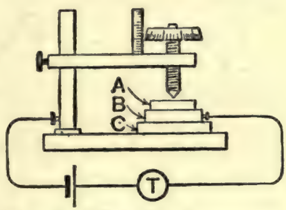

Fig. 3.—Electric micrometer.

Note.—See that the point where electric contact is made by the screw is clean.

Exercise.—Determine the thickness of the small metal discs provided.

3. Spherometer.—Find the values of the divisions of the linear and circular scales of the spherometer. Then find the "zero" of the instrument as follows. Place it on the plane glass plate provided and turn the screw until its point just meets the image of the point reflected at the glass surface. Read the scales. This gives the *zero*. Turn back the screw. Insert the object, of which the thickness is required, below the

* Due to Dr. P. E. Shaw of Nottingham.

screw point. Turn down the screw until the point just touches the object. Generally the surface of the object is such that an image of the point of the screw is obtained by reflection, and the point and its image are made just to meet as before. Read both scales and so determine the thickness of the object.

EXERCISE.—Determine the thickness of the small metal discs provided.

3. Radius of Curvature.

APPARATUS.—Spherometer, Plane Glass Plate, Steel Rule, two or more Spherical Surfaces.

OBJECT.—To find the radius of curvature of a spherical surface, convex or concave.

PRELIMINARY.—The spherometer is so called, because, by means of it, the radius of a spherical surface may be found, although only a part of the spherical surface is available. For instance, the radius of the surface of a lens or of a convex or concave mirror is easily determined by its means.

THEORY AND METHOD.—First take the "zero" reading of the spherometer, as in Experiment 2, by placing it on the plane glass surface. Then place the spherometer on the curved surface and turn the screw until its point is just touching the surface. Take the reading.

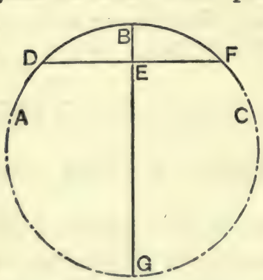

FIG. 4.—Theory of spherometer.

In Fig. 4 let ABC be the spherical surface and let D be one foot of the spherometer, as it rests on the surface, and B the point where the screw touches the surface. Let DEF be the trace of the plane perpendicular to the paper, in which the three feet of the spherometer touch the surface. Obviously E is the *zero* position of the point of the screw and DE is the distance from one foot to the point of the screw in this position (F, it will be noticed, is *not* a point, where any foot rests).

The distance BE is the difference between the two readings of the spherometer, which have been taken. From DE and BE the radius R of the surface is easily obtained.

For, complete the circle ABC and produce BE to G,

$$\text{Then BE . EG} = \text{DE . EF} = \text{DE}^2$$
$$\therefore \text{BE}(2\text{R} - \text{BE}) = \text{DE}^2,$$

from which R may be determined.

To measure DE, press the spherometer in the *zero* position on to a sheet of paper and measure the distance between the impress of each foot and the impress of the screw and take the mean of the three readings.

Instead of using the distance between one foot and the screw, it is generally advised to use the distance between two feet of the spherometer, as this distance is larger, and may, therefore, be measured to greater accuracy. If this is done, it is necessary to express DE, in the above formula, in terms of the distance between two feet.

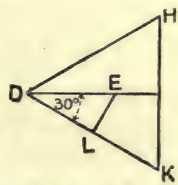

Fig. 5.—Plan of spherometer feet.

Let DHK in Fig. 5 be a plan of the spherometer feet and L the middle point of DK.

$$\text{Then DL/DE} = \cos 30^\circ = \sqrt{3}/2$$
$$\therefore \text{DE} = 2\text{DL}/\sqrt{3} = \text{DK}/\sqrt{3}$$
$$\therefore \text{BE}(2\text{R} - \text{BE}) = \text{DK}^2/3$$
$$\therefore \text{R} = \text{DK}^2/6\text{BE} + \text{BE}/2.$$

This formula is generally written

$$\text{R} = l^2/6a + a/2$$

where $\text{DK} = l$, and $\text{BE} = a$.

Note, when a is very small

$$\text{R} = l^2/6a \text{ nearly}$$

Exercise.—Measure the radii of curvature of the given convex and concave surfaces.

The student must be able to reproduce the theory given above.

4. Optical Lever.

APPARATUS.—Glass Plate, Wood Block, Vertical Scale, Metre Scale, Cover-slips, Lamp and Optical Lever with Concave Mirror (or Telescope and Optical Lever with Plane Mirror).

OBJECT.—To measure a small distance by a tilting mirror.

THEORY AND METHOD.—In Fig. 6, let B be the back foot and C the line of the front feet of the optical lever.

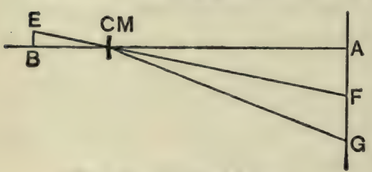

FIG. 6.—Optical lever.

Light from the lamp is focussed by the concave mirror, M, at A on the scale, AG. A definite point of the image is taken and its position on the scale noted. The glass cover-slip is now slipped carefully under the foot B, so that the foot rises to E. ECF is now the line of the optical lever and the image of the lamp is now obtained at G, the angle AMF being equal to the angle FMG.

Then $EB/BC = AF/AC$. Measure BC, the perpendicular distance from the back foot to the line of the front feet. Measure AC, the distance from the front feet to the scale. AF is very approximately $AG/2$, since the angles AMF and GMF are very small and equal. Thus the unknown thickness EB may be found.

EXERCISE.—Find the thickness of the given cover-slip and compare the thickness thus found with the thickness found by a screw gauge.

USE OF TELESCOPE.—Sometimes the optical lever has a plane mirror instead of a concave. In that case a telescope is placed at A and focussed on to the image of the scale in the mirror. The scale should be well illuminated. The division of the scale, which coincides with the cross-wire of the telescope, is read. The cover-slip is slipped under foot B, and the scale division again read. From what has been said, it

will be evident how these two readings, together with the lengths BC and AC, enable the thickness of the cover-slip to be determined.

EXERCISE.—Find in this way the thickness of the given cover-slip.

RESULTS.—Explain, with sketches where necessary, what you have done and observed, tabulating neatly your observations and conclusions.

5. Level Surface.

APPARATUS.—Levelling Instrument, Telescope with cross-wires, Dish, Plane Glass Plate, Rule, Levels, Small Plane Mirror.

OBJECT.—To set a given glass plate quite level.

METHOD.—The dish is partly filled with water and placed on the levelling apparatus. The levelling apparatus consists of a metal plate supported on three legs. From the upper surface of the plate project three metal points, A, B, C (see Fig. 7), which serve as supports for anything placed on them.

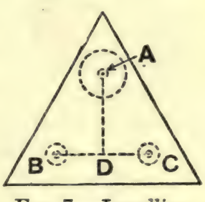

FIG. 7.—Levelling apparatus.

Two of these points, viz. A and B, are the points of screws passing through the plate. One of these screws, A, is fitted beneath the plate, like a spherometer, with a linear scale and large head with circular scale. An image of some *distant* object is obtained in the telescope by reflection from the surface of the water in the dish. The telescope is adjusted until a certain point of the image is at the intersection of the cross-wires. The glass plate is then substituted for the dish and the two screws, A and B, adjusted until the same point of the image comes again to the intersection of the cross-wires. Then the surface of the glass plate, if truly plane, should be perfectly level.

EXERCISE 1.—Now place the levels on the glass plate and see if they are accurate. If they are inaccurate find what angular movement of the plate is required to bring the bubble

to the centre. This is done as follows: Place the level along the line AD, where D is the middle point of BC as indicated in the diagram. Read the circular scale on the head of the screw A. This is conveniently done by placing a mirror on the table under it. Now turn the screw A until the bubble comes to the centre. Read the circular scale again. The difference between these two readings divided by the distance AD will give the required angle in radians. Express this angle in minutes and seconds.

Exercise 2.—Find the sensitiveness of the levels, which may be measured by the angle through which the level has to be turned to give 1 mm. displacement of the bubble. This angle is found exactly as above. Express it in minutes and seconds.

6. Graphs.

Apparatus.—Squared Paper, Ruler.

Object.—To plot curves, called *graphs*, which show the relation between two quantities connected in a given manner.

Definitions.—That quantity to which any value may be assigned is called the *independent variable*, that quantity whose corresponding value then follows as a consequence of the relation between them is called the *dependent variable*. In the diagram to be drawn the various values of these two quantities are measured off along two lines at right angles to each other called *co-ordinate axes*, their point of intersection being called the *origin*. Distances measured along the horizontal axis are called *abscissæ*, and distances along the perpendicular axis are termed *ordinates*, *positive* quantities being measured to the right and upward from the origin, *negative* ones to the left and downward.

Method.—From the given relation, tabulate in two columns the values of one quantity and the corresponding values of the other. Choose scales for the two axes such that the extreme values of the quantities will come on the paper. Measure a value of the independent variable along the axis of abscissæ and the corresponding value of the dependent variable along

the axis of ordinates. Draw (or imagine to be drawn) from each point thus determined lines perpendicular to the axes on which they are situated. The intersection of these perpendiculars gives one point on the curve desired. In like manner obtain a series of points and draw the curve so as to pass through the various points.

EXERCISES.—Plot curves with the variables as given below.

Independent variable.		*Dependent variable.*
(*a*) Side of square.		Area of the same square.

From the curve obtained find the square roots of 30 and of 60. Look in a book of tables and see if your results are correct.

(*b*) Side of a cube.		Volume of the same cube.

From the curve find the cube roots of 100 and of 600.

(*c*) Length	and	Width of a rectangle of constant area.

(*d*) Determine by a graphical method the length and width of the largest possible rectangle, the sum of whose sides is 72 feet.

7. Volumes and Heights.

APPARATUS.—Jars, Beakers and Flasks of various shapes, Steel Rule, Small Test Tube, Cork and Knitting Needle.

OBJECT.—To find the law connecting the *volume* of a liquid in a vessel and the *height* to which it fills that vessel.

METHOD.—Press the needle perpendicularly into the cork, and, resting the latter on the mouth of one of the given flasks, make the end of the needle just touch the bottom of the flask. Take out the needle and cork, measure the length of the needle below the cork. Fill the test-tube with water and empty into the flask. Bring the end of the needle in contact with the water surface and again measure the length of the needle below the cork, and proceed in this way to obtain a series of heights and corresponding volumes. A curve, called a *calibration* curve, having volumes as ordinates and heights as abscissæ, may then be plotted.

Exercise.—Predict by a curve on paper the general form of the curve for some of the other vessels not yet used. Then perform the experiment and plot the actual curve, and note the agreement or discrepancy between the two curves.

Result.—Describe what you have done and plot the calibration curve of each vessel. How from the calibration curve would you determine the internal cross-section of the vessel at any specified height?

8. Uniform Speed.

Apparatus.—Atwood's Machine, Clock beating seconds.

Object.—To ascertain whether or not the equal masses on the machine move with uniform speed after removal of the overweight.

Method.—Having ascertained that the two masses exactly balance, place an overweight on the right-hand mass and arrest it at the top. Arrange the ring and the platform so that, on starting the masses at one tick of the clock, the overweight is removed by the ring at the second tick, and the mass arrested by the stage at the third tick of the clock. The distance between the ring and the stage (*less the depth of the weight*) will thus give the space passed over in one second after removal of the overweight, and hence expresses the velocity possessed by the masses during the second in question. Now, leaving the ring where it is, lower the stage until two seconds elapse between the removal of the overweight by the ring and the arrestment of the weight by the stage. Thus find the average velocity possessed by the masses during these two seconds.

Proceed in the same way, making the time between the removal of the overweight and the arrestment of the weight by the stage, 3, 4, etc., seconds. Calculate the mean velocity during each period and observe whether the velocity is uniform or not.

Results.—Give your results in tabular form and plot a curve connecting times with distances after removal of the

overweight. If the curve is a straight line, the velocity is uniform. If the curve is not a straight line, what features of the curve represent

(*a*) the mean velocity during any period?

(*b*) the velocity at any instant?

9. Uniform Acceleration.

APPARATUS.—Atwood's Machine, Clock beating seconds.

Part I.

OBJECT.—To ascertain that unequal masses on the machine move with uniform acceleration and to determine the acceleration.

METHOD.—See that the two masses, before the overweight is added, balance exactly. Then test, as in the previous experiment, to see if, when once the two equal weights are started, they continue to move with uniform velocity. If they do not, add small pieces of wire until they do. Then place the overweight on the right-hand side.

Arrange the ring and the platform so that, on starting the masses at one tick of the clock, the overweight is removed by the ring at the second tick and the mass arrested by the stage at the third tick of the clock. Hence, calculate the velocity possessed by the masses at the instant when the overweight was removed, *i.e.* the velocity acquired during one second of acceleration, which, of course, equals numerically the acceleration itself.

Now arrange the ring and stage so that two seconds elapse from the start until the instant when the overweight is removed by the ring, and one second more until the masses are arrested by the stage. Hence, calculate the velocity acquired during two seconds of acceleration and thus find the acceleration itself.

Proceed in this way, finding the velocities acquired during 3, 4, etc., seconds of accelerated motion, and deducing from each velocity the value of the acceleration itself. Note that in each case the depth of the weight must be subtracted from the distance between the ring and the stage.

RESULTS.—Plot a curve connecting times from rest to removal of overweight with velocities acquired during these times. If the curve is a straight line, the acceleration is uniform.

Express your results in tabular form thus—

Time from rest to removal of overweight.	Distance from ring to stage.	Velocity acquired in cms. per sec.	Acceleration in cms. per sec. per sec.

Part II.

OBJECT.—Suppose the ring removed, and thus the overweight to remain on one of the masses during the whole time of motion. Then, if s is the distance passed over from start to finish in time t, it may be shown theoretically that $s = \frac{1}{2}at^2$, where a is the acceleration. The object of the experiment is to show that s is proportional to t^2 and thus confirm the above relation. Also the acceleration is to be determined by means of this relation.

METHOD.—Using the same masses and overweight as in Part I, arrest the larger mass at the top, remove the ring entirely and place the stage so as to arrest the masses after the lapse of one second from the start. Note the distance s_1 passed over. Now let two seconds elapse from the start before the masses are arrested by the stage. Note the distance s_2 passed over. In this manner obtain the distances s_1, s_2, s_3, s_4, etc., passed over in 1, 2, 3, 4, etc., seconds of accelerated motion. In this case the depth of the weight does not come into consideration. Calculate the acceleration a in each case.

RESULTS.—Tabulate your results in columns headed s, t, t^2, s/t^2, a. If a is a constant then s is proportional to t^2.

Plot a curve connecting s with t^2. If the curve is a straight line, then s is proportional to t^2.

10. "g" by Atwood's Machine.

Apparatus.—Atwood's Machine, Clock, beating seconds.

Object.—To prove that the acceleration a is proportional to the mass of the overweight m divided by M the total mass moved, and from this to deduce the value of g.

Method.—Using, say, the second method given in the previous experiment, find the accelerations for three different pairs of masses and different overweights. If the above relation between the acceleration and the masses is true, then $a \div m/M$ is a constant.

Results.—Show that this is the case, tabulating your results in columns headed, m, M, a, $a \div m/M$.

When a body falls freely under gravity, its acceleration is denoted by g. But in this case $m = M$. Thus $a = g$, when $m = M$. And since the experiments show that a is proportional to m/M, we see that a must be equal to $g\ m/M$. So that the values in the last column are equal to g. Find the mean of these values.

11. "g" by Simple Pendulum.

Apparatus.—Simple Pendulum, Metre Rule, Watch with seconds hand. (A telescope may be used.)

Objects.—*First*, to prove what effect the various factors (1) length of pendulum, (2) amplitude of swing, and (3) mass of the bob have on the period of oscillation of the pendulum, and *second*, to determine g.

Exercise 1.—*Relation between length of pendulum* and *period of oscillation.*

The period of complete oscillation, T, is the time taken for the pendulum to pass from the extreme position on one side over to the other side and back again to its original position. If l, is the length of the pendulum, then T^2 is proportional to l, that is, l/T^2 is constant, whatever the length of the pendulum, provided the amplitude of swing is very small. For a length of a metre a movement of the bob of about one or two

cms. to each side of its position of rest is ample. You are required to prove by experiment the above relation between l and T.

If a standard clock is at hand, check the watch against it by comparison of the indications of the seconds hands over a period of five minutes. If any error is found, a correction must be applied throughout the experiments below.

Vary the length of the pendulum and determine the periods for, at least, four different lengths. The length of the pendulum is to be measured from the point of support to the centre of the bob. In order to get the period of oscillation accurately, the time of 100 oscillations should be taken at each determination. Calculate l/T^2 and tabulate your results in columns headed, l, T, l/T^2, and so find the mean l/T^2.

RESULT.—Plot a curve connecting l with T^2, and observe whether the above-mentioned relation holds.

EXERCISE 2.—*Influence of amplitude of swing.* Keeping the length of the pendulum and mass of bob constant, vary the amplitude. Find the corresponding periods. Tabulate your results and draw your conclusion.

EXERCISE 3.—*Influence of mass of bob.* Keeping the length of pendulum and the amplitude of swing constant, use bobs of various sizes and find the corresponding periods. Tabulate results and draw your conclusion.

EXERCISE 4.—If g is the acceleration due to gravity, the equation connecting T, l, and g, provided the amplitude is small, is $T = 2\pi\sqrt{l/g}$.

Thus $l/T^2 = g/4\pi^2$. Now l/T^2 has been determined above, and its mean value stands in the last column of the table in Exercise 1. Using this mean value, find g.

RESULT.—Express g both in cm.-sec. units and in foot-sec. units.

12. Blackburn's Pendulum.

Apparatus.—Blackburn's Pendulum, Metre Scale.

Object.—To find the path of a point which executes two vibrations at right angles.

Method.—The bob of the pendulum consists of a heavy metal ring, B, in which rests a funnel with a fine stem. The ring is hung from two strings fixed to rings, D and E, which slide on a horizontal bar.

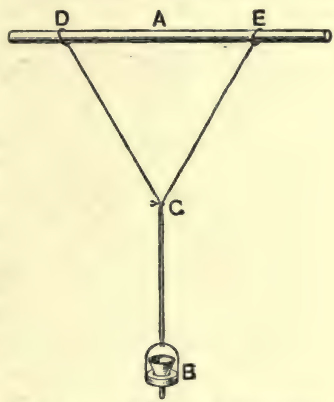

Fig. 8.—Blackburn's pendulum.

Up and down the strings a little loop of wire, C, can slide. The arrangement now consists of two pendulums, one of which, AB, swings in a plane perpendicular to the paper, while the other, CB, may be made to swing in the plane of the paper. Thus, by starting the bob from a suitable point, it may have at the same time two periodic motions at right angles to each other. The path of the bob is then the result of these two motions.

The path is recorded by filling the funnel with fine sand and allowing the sand, as it runs out, to make a trace on a blackboard placed under the pendulum. By sliding C up and down, the relative lengths of the two pendulums may be altered and the ratio of the periods of oscillation of the two pendulums may be given any value.

Exercise 1. *Ratio of periods* 1 : 1.—If D and E are slid together, the two pendulums have the same length and consequently the same periods. It will be found that the bob, according to the way it is started, will describe an ellipse, a straight line or a circle. See the first line of Fig. 9.

Exercise 2. *Ratio of periods* 2 : 1.—If C is so placed

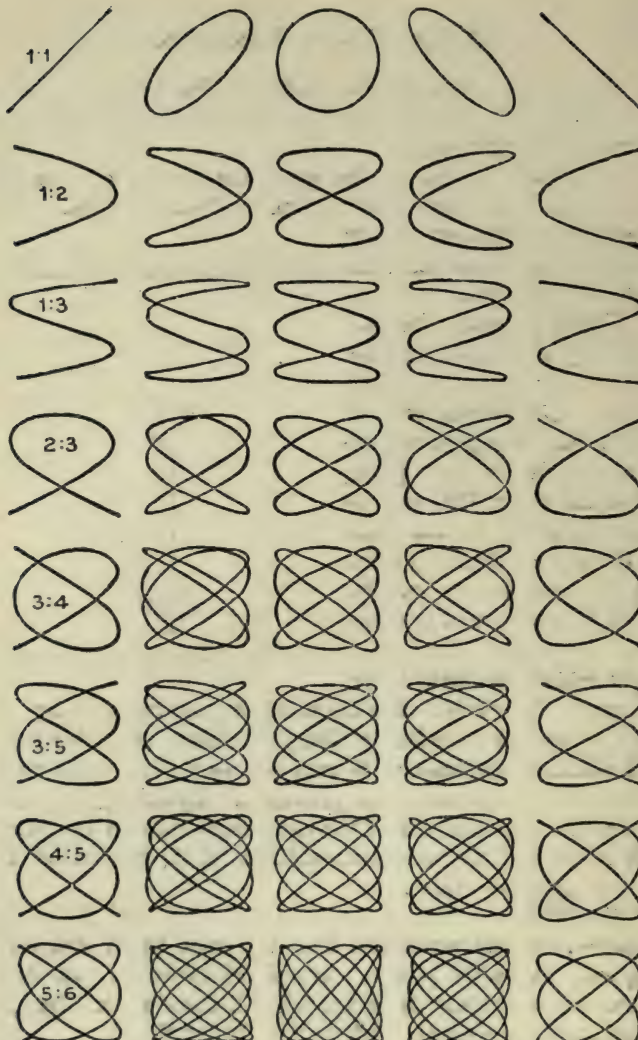

FIG. 9.—Resultants of rectangular vibrations.

that AB : CB is 4 : 1; then, from the experiment on the simple pendulum, it is evident that the periods are in the ratio of 2 : 1. Find the figures which the bob describes. It will describe the figures in the second line of Fig. 9. Sketch the figures.

Exercise 3. *Ratio of periods* 3 : 1.—Make the ratio of length AB to length CB = 9 : 1. Then the ratio of the periods is 3 : 1. Sketch the various figures. See the third line of Fig. 9.

Exercise 4. *Ratio of periods* 3 : 2.—The student will understand how to proceed. See the fourth line of Fig. 9.

Exercise 5. *Ratio of periods* 4 : 3.—See the fifth line of Fig. 9. Sketch the figures in each case.

Stand so as to face the plane of strings DCE. Count the number of complete swings in plane DCE to complete a figure, *i.e.* from the time the bob leaves a point till it reaches it again. Stand facing along the plane DCE and count as before. The following law will be observed. The number of complete swings in a given time in one plane will be to the number in the other plane in the ratio of the frequencies of the two simple pendulums. Thus the experiment teaches that, given any figure, which has been obtained by the combination of two periodic motions at right angles, it is possible to tell at once from the figure the ratio of the periods of the two component motions.

13. Density by Simple Method.

Apparatus.—Strong Balance, Weights, Substances regular and irregular in shape, Steel Rule, Calliper Gauge, Displacement Apparatus, Measuring Jar, Liquids of which the densities are to be determined.

Definition.—The density of a substance is the quotient (mass ÷ volume), and so may be defined as the number of grams of the substance per 1 c.c.

Method for Solid.—Find the mass of the substance by weighing. Then find the volume of the substance. In the

case of a solid, if the body be regular in shape, find its volume from its dimensions. If, however, it is not regular in shape, find its volume by means of the displacement apparatus shown in Fig. 10.

Fill the apparatus with water up to the mark A on the index tube. Place the substance in the water and run the water out into the measuring jar until it stands at the same mark A as before. See that the substance is still completely covered with water. The amount of water in the measuring jar gives the volume.

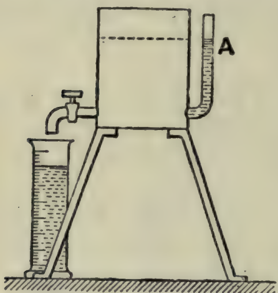

Fig. 10.—Displacement apparatus.

Method for Liquid.—To determine the density of a liquid, find the mass by weighing and the volume by the measuring jar. The difficulty lies in obtaining the volume correctly, so take as much liquid as possible, since this reduces the percentage error.

Results.—Give the densities of the solids and liquids provided. Use different quantities of each solid and liquid and plot graphs with volumes as abscissæ and masses as ordinates. What shape should the curve for any one substance be and what does its slope denote ?

14. Sensibility of a Balance.

Apparatus.—Delicate Balance, Box of Weights.

Object.—The sensibility of a balance may be measured by the change in the balance point when a 10 mg. weight is placed in one pan of the balance. The sensibility varies with the load in the pans, and the object of the experiment is to determine the sensibility for various loads.

Method.—To obtain the balance point proceed as follows in every case. Set the balance swinging gently, close the front

to shield the balance from air currents, and make a series of readings of consecutive turning points of the pointer on both sides, as in Table I.

TABLE I.

Turning Points Observed.

6·5	19·6
7·5	
8·2	18·9

Then calculate the mean of two consecutive readings on the same side, and from this mean and the corresponding reading on the other side, calculate the balance point as shown in Table II. Finally obtain the mean balance point as shown at foot of Table II.

TABLE II.

Calculation of Mean Balance Point.

6·5 (7·0)	13·3	19·6
7·5	13·4	(19·3)
(7·9) 8·2	13·4	18·9

13·4 is Mean Balance Point.

The reasons for taking readings while the balance is swinging, are—

(1) Time is saved.

(2) The effect of friction at the point of support of the balance is overcome. Now, the amplitude of the swing is gradually falling off owing to air resistance, and the above method of obtaining the balance is adopted, because it would obviously be incorrect to simply take the mean of two consecutive turning points, one on each side.

EXERCISE.—Obtain the sensibility of the balance for various loads as follows :—

(1) With no weights in the pans. Place a 10 mg. weight in one pan, free the balance from the arrestment, and find the balance point as above. Arrest the balance, transfer the 10 mg.

weight to the other pan, free the balance, and again find the balance point. Half of the difference in the balance point readings gives the sensibility.

(2) With 20 grams in each pan. Place a 20 gram weight in each pan and a 10 mg. weight in one pan. Free the balance and find the balance point. Transfer the 10 mg. weight to the other pan and again find the balance point. From the balance point readings find the sensibility.

(3) Proceed exactly as above with 40, 60, 80, 100 grams in each pan, and find the sensibility in each case. Tabulate your results as in Table III.

TABLE III.

Calculation of Sensibility.

Load in one pan.	Balance points.		Sensibility.
	10 mg. in right-hand pan.	10 mg. in left-hand pan.	
0	7·1	13·3	3·1
20	6·9	13·7	3·4
40	7·0	13·4	3·2

Results.—Draw a curve connecting the sensibility with the load.

Note.—The usefulness of this curve is that, when once it has been obtained, a body may be weighed accurately to 1 mg., even though no rider is provided. For, corresponding to any load, the curve gives the shift of the balance point for 10 mg. The weight, therefore, required to produce any smaller shift than this may then be calculated by proportion, and thus the weight of a body may be obtained correct to the third decimal place of a gram.

15. Correct Work with False Balance.

Apparatus.—Delicate Balance, Box of Weights, Body to be Weighed.

Object.—To obtain the weight of a body in spite of the fact that the arms of the balance may not be absolutely equal in length, and to obtain the ratio of the lengths of the arms of the balance.

Two methods may be used to obtain the weight of a body.

Method I. *With Counterpoise.*—Place in the left-hand pan a counterpoise slightly heavier than the body to be weighed. Balance the counterpoise by weights in the right-hand pan, obtaining the balance point as in the previous experiment. Having noted the weights, remove them. Now place the body, of which the weight is required, in the right-hand pan, and add weights until the same balance point as before is obtained.

In general it will not be possible to obtain this balance point accurately by means of the 10 mg. weight in the box. A smaller weight still may be necessary. A 10 mg. rider is sometimes provided for this purpose. It is placed on the graduated arm of the balance, and when situated at division 1, it is equivalent to 1 mg., at division 3, to 3 mg., and so on. If no rider is provided, find the shift of the balance point when a 10 mg. weight is added to the weights, and calculate by proportion the weight which would give the balance point required. The difference of the two sets of weights in the right-hand pan will obviously be the weight of the body, even though the arms of the balance are not equal in length.

Method II.—*By Interchange.*—Find the balance point of the balance as in the previous experiment. Place the body in the right-hand pan and balance with weights in the left-hand pan to this balance point. If no rider is provided, it may be necessary to proceed as in Part I. above to obtain the balance point correctly. Let the weight be w_1. Now remove the

weights and place the body in the left-hand pan and balance it with weights in the right-hand pan, as before, to the same balance point. Let the weight be w_2. Then if x is the weight of the body, and a, b are the lengths of the arms of the balance, we have—

$$ax = bw_1, \quad bx = aw_2$$
$$\therefore x = \sqrt{w_1 \times w_2} \text{ and } a/b = \sqrt{w_1/w_2}.$$

Thus the true weight of the body and the ratio of the arms of the balance are found.

Note that, since w_1 and w_2 are very nearly the same, $\sqrt{w_1 \times w_2} = \frac{1}{2}(w_1 + w_2)$ very approximately.

16. Density of a Liquid by Bottle.

APPARATUS.—Specific Gravity Bottle, Delicate Balance, Weights, Liquids of which the density is required.

PRELIMINARY.—The fact that 1 gram of water has a volume of very nearly 1 c.c. at ordinary room temperatures may be made use of to obtain the volume of a liquid or solid, and so to determine densities. In this experiment it is employed to obtain the density of a liquid.

METHOD.—Find the mass of the liquid required to fill the specific gravity bottle by weighing the bottle empty and then full of the liquid. Find the internal volume of the bottle, and thus the volume of the above mass of liquid, by weighing the bottle full of water, and subtracting the mass of the bottle when empty. In this way find the densities of the given liquids.

NOTE.—If the bottle is dirty, clean it as follows. Wash it with caustic soda, distilled water, nitric acid, and distilled water, and dry by drawing a current of hot air into the bottle by an air-pump or otherwise.

RESULTS.—The densities of the given liquids.

17. Density of a Solid in Small Pieces.

APPARATUS.—Balance, Weights, Specific Gravity Bottle, Watch Glass, Solid in small pieces (such as lead shot, sand, etc.), Screw gauge.

PRELIMINARY.—The fact that 1 gram of water has a volume 1 c.c. is used in this experiment to find the volume of a certain quantity of a solid, and so to find the density of a solid in small pieces.

METHOD.—Weigh out a certain quantity of the substance on a watch glass. To obtain the volume of this quantity proceed as follows. Let the substance be one which does not dissolve in water, say, lead in the form of shot. Fill the specific gravity bottle with water and place the bottle and the watch-glass with the substance on it in one pan of the balance. Counterpoise these with weights or lead shot. Now put the substance into the bottle and wipe off all traces of the water, which has been expelled. Replace the bottle and watch-glass on the pan, and to this pan add weights until the balance is restored. Obviously the weight in grams added is the weight of the water which has been expelled by the substance, and thus, numerically, is the volume of the substance in c.c. The mass of the substance divided by its volume gives the density required.

RESULTS.—Find in this way the densities of lead and the other substances provided.

Find also the mean volume of one lead shot, and from this mean diameter of the lead shot. Check this result by finding the mean diameter of twenty of the shot by means of the screw-gauge.

NOTE.—If the substance dissolves in water, find as above its density relative to some liquid in which it does not dissolve, and then the density of this liquid relative to water by the bottle, and hence the density of the substance relative to water.

18. Liquid Pressure.

Apparatus.—Several Cylindrical Tubes closed at one end and differing in diameter. Jars to contain Water, Weights, Steel Rule, Lead Shot, Liquids of known density (or Salt to make a Solution, and Balance and Liquid Measure to determine its density).

Object.—To investigate the force exerted by a liquid on a plane surface on which it presses.

Preliminary.—The plane surfaces in this case are the flat ends of the closed cylindrical tubes. The pressure which the liquid exerts on such a surface depends on (1) the depth to which the tube is sunk, (2) the area of the end, (3) the density of the liquid. There are thus three conditions on which it depends ; take care to vary only one at a time.

Exercise 1 : *Depth.*—Using always the same tube, sink it to various depths by putting lead shot inside it. In each case note the depth and the total weight of tube and contents. This weight is equal to the total pressure exerted by the liquid on the base of the tube. Tabulate depths and weights. Plot a curve and deduce the relation between *depth* and pressure.

Exercise 2 : *Area.*—Using tubes of different cross-sections sink them all to the same depth. Determine cross-sections and total weights in each case. Tabulate cross-sections and weights. Plot a curve and deduce the relation between *area* and pressure.

Exercise 3 : *Density.*—Using liquids of different densities, sink the same tube to the same depth in each. Note the total weight of tube in each case, and, if necessary, determine the densities of the liquids. Tabulate densities and weights. Plote a curve and deduce the relation between *density* and pressure.

Result.—Summarise the conclusions to which your experiments have led you.

19. Densities by U-Tube.

Apparatus.—U-Tubes erect and inverted, Clip Stand, Steel Rule, Beakers, Liquids whose densities are to be determined.

Object.—To obtain relative densities of liquids by U-tube, erect or inverted.

Method 1: *Erect* U-*Tube.*—If the liquids, of which the densities are to be compared, do not mix, pour one liquid down one side of the U-tube and the other down the other. Measure the heights h_1, h_2 from the place where the liquids meet to the upper surfaces of the liquids. Then it should be evident from the preceding experiment that $h_1 d_1 = h_2 d_2$, where d_1 and d_2 are the densities of the two liquids. (If the student cannot show that this is so, he must ask a demonstrator.) Thus $d_1/d_2 = h_2/h_1$. If the one liquid is water the density of the other liquid is obtained absolutely, since the density of water is *unity*.

Note.—For the sake of accuracy make h_1 and h_2 as large as possible. If two liquids mix, use a third liquid to separate them which does not mix with either. The student should have no difficulty in knowing how to proceed.

Method 2: *Inverted* U-*Tube.*—It is convenient to use the inverted U-tube, especially when two liquids mix. The limbs of the U-tube are arranged to dip in the liquids in the beakers, one limb, say, in water and the other in the liquid of which the density is required. The beakers should be nearly full. Draw the liquids up into the tubes and measure the heights, h_1 h_2, from the surface of the liquids in the beakers to the surface inside the tubes. As above, $h_1 d_1 = h_2 d_2$ (and this the student should be able to prove). Thus, as before, $d_1/d_2 = h_2/h_1$, where d_1, d_2 are the densities of the liquids.

Exercise.—Draw the liquids to different heights and tabulate your results in columns headed liquid 1, liquid 2, h_1, h_2, relative density: $d_1 \div d_2 = h_2 \div h_1$. If liquid 2 is water the last column gives us the density of the other liquid.

RESULTS.—Tables of results with sketch of apparatus. Relative densities or absolute densities of the liquids provided.

20. Archimedes' Principle.

APPARATUS.—Strong Balance, Brass Cylinder with Case, Lead Shot, Box of Weights, Bridge, Beaker, Pipette, Bodies of which Volume is required (Delicate Balance, Thin Iron Wire, Metre Scale).

PRELIMINARY.—The *principle of Archimedes* states that, when a body is immersed in a liquid, the resultant of the liquid pressures experienced by it is an upward force equal to the weight of the liquid displaced by it. Thus a body immersed in a liquid apparently loses weight, and this loss in weight is equal to the weight of the liquid which is displaced.

OBJECT.—To confirm this classic principle and apply it.

METHOD.—The apparatus consists of a brass cylinder, which fits exactly into a case made of thin brass. The volume of the cylinder and the internal volume of the case are thus, as nearly as possible, the same. The case can be hung from the beam of a balance, and a hook projects from its bottom from which the cylinder may be suspended.

Hang the case on the beam of the balance and the brass cylinder on the hook at the bottom of the case. Counterpoise with lead shot. Surround the cylinder with a beaker, which stands on a platform over the pan of the balance, and fill up the beaker with a liquid until the cylinder is completely covered. The case and cylinder now appear to weigh less than before. Now fill the case with the liquid by means of the pipette, and it will be found that balance has been restored. Thus the loss in weight of the cylinder, when immersed, is equal to the weight of a quantity of the liquid which has the same volume as the cylinder ; *i.e.* is equal to the weight of the liquid displaced.

This principle gives a means of finding the volume of a body. For, since 1 gram weight of water occupies very nearly

1 c.c. at ordinary temperature, the loss in weight of a body in grams, when it is immersed in water, is numerically equal to the volume of the water displaced in c.cs., that is, is numerically equal to the volume of the body.

Exercises.—Find, in this way, the volumes of the bodies provided. (Find also, by means of a delicate balance, the mean diameter of a piece of thin iron wire.)

21. Variable Hydrometers.

Apparatus.—Hydrometers of variable immersion with Liquids and Jars suitable to each. Salt, Balance and Weights, Liquid Measure.

Preliminary.—Since a body immersed in a liquid apparently loses weight equal to the weight of the liquid displaced, the weight of the liquid displaced by a *floating* body must be equal to the weight of the body. Thus a floating body sinks until it displaces its own weight of liquid, and since this depends upon the density of the liquid, the amount to which the body sinks may be made a measure of the density of a liquid. On this principle hydrometers of variable immersion are graduated to give the densities of liquids in which they float, from the graduations to which they are immersed.

The methods of graduation, however, vary according to the liquids for which the instrument is designed. The meanings of the graduations on any special instrument must therefore be acquired from its maker or ascertained by careful tests.

Exercise.—Make solutions of salt by adding salt, five grams at a time, to a known quantity of water. Find the density of each solution by a hydrometer and draw a curve connecting density with the amount of salt present in the solution. (Determine the density of a solution by another method and so check the accuracy of a particular reading of the hydrometer.)

22. Nicholson's Hydrometer.

Apparatus.—Nicholson's Hydrometer, Jar, Weights, Lead, Wax, and other Solids.

Object.—To determine the densities of solids by Nicholson's hydrometer.

Preliminary.—This hydrometer differs from those of the previous experiment in that it is always adjusted until immersed to the same point. Thus the volume, and consequently the weight, of the displaced liquid is always the same. To displace this weight of liquid the hydrometer is sunk by placing weights on it, and it is obvious that in this way bodies may be weighed. This hydrometer is, therefore, used to obtain the weight of a body in grams, first in air, and secondly when it is immersed in water. The loss of weight after immersion is thus obtained, and this in grams is numerically the same as the volume of the body in c.cs. (See Experiment 20.) Thus the mass and volume of the body are determined, and so the density follows as their quotient.

Method.—Sink the hydrometer to the mark on the stem by placing weights W_1 gms. on the upper pan. Remove these weights, and now place the body on the upper pan, add weights W_2 gms. until the hydrometer is sunk to the mark. Obviously $W_1 - W_2$ gives the mass of the body. Now place the body in the lower pan and let the weight in the upper pan necessary to sink the hydrometer to the mark be W_3 gms. Obviously $W_3 - W_2$ is the apparent loss of weight of the body on being immersed in water, and this is numerically equal to its volume in c.cs. Thus the density of the body is $(W_1 - W_2)/(W_3 - W_2)$.

Note.—If the body floats in water it will be necessary to tie it to the bottom pan. Note also that in making the adjustments great care must be taken to attain all possible accuracy, for the hydrometer is not very sensitive, and a slight error in the weights in the pan will therefore cause

a very marked deviation from the correct result. Further, an error of 1 per cent. in W_3 or W_2 may cause an error of 10 per cent. in $W_3 - W_2$, and therefore an error of this amount in the value found for the density.

RESULTS.—Densities of bronze and silver coins, wax, etc.

23. Density of a Solid that sinks in Water.

APPARATUS.—Delicate Balance, Weights, Bodies that sink in water, Thread, Beaker, Platform over pan of balance.

METHOD.—Find the mass by weighing and find the volume by the principle of Experiment 20. Thus determine the density.

If it is not known that the arms are equal it will be well to use the counterpoise method of weighing (see Experiment 15), and to proceed and note the results as follows :—

(1) Counterpoise in left pan balanced by W_1 gms. in right pan
(2) " " " body in air + W_2 "
(3) " " " body in water + W_3 "

It is then evident from the theory above, that we have

$$(W_1 - W_2)/(W_3 - W_2) = \text{density of a body in gms. per c.c.}$$

NOTE.—(1) The volume occupied by 1 gram of water is 1 c.c. only at 4° C. In general, the temperature of the water used in the experiment will not be 4° C. For accurate work, therefore, the temperature of the water must be observed, and the volume occupied by 1 gram of water at this temperature found from tables, and from this the volume of the body must be calculated.

(2) Bubbles of air must not adhere to the body, when placed in the water. The bubbles may be removed by a wire. It is advisable to use water from which the air has been expelled by boiling.

(3) The "*specific gravity*" of a substance is the weight of the substance divided by the weight of an equal volume of water at 4° C. Thus, numerically, the "*specific gravity*" of a substance is the same as its density in *c.g.s.* units. But the former will be a number, while the latter is in gm./c.c. In

other units than the c.g.s. system the *density* may be denoted by a number quite different from that of the *specific gravity*. Thus cast iron has a *specific gravity* 7·25, a *density* 7·25 gms./c.c., or 0·26 lb./cu. in.

(4) If the substance dissolve in water, we may find as above the specific gravity of the substance relative to some liquid in which it does not dissolve. Then find the specific gravity of this liquid relative to water by one of the methods given subsequently. The product of these two specific gravities will give the specific gravity of the substance relative to water, or its density in gms. per c.c.

RESULTS.—Record everything done and observed, and state the densities of the substances provided. Check these results by means of tables of densities.

24. Density of a Solid that floats in Water.

APPARATUS.—Delicate Balance, Weights, Bodies that float in water, Thread, Beaker, Sinker, Platform over pan of balance, Lead Shot.

METHOD.—The method is the same as in the previous experiment. But, as the body floats in water, it is necessary to use a sinker to immerse it completely. A good plan is to have the sinker in the form of a cage, inside which the body may be placed. Proceed as follows : Set the beaker of water on the platform over, say, the right-hand pan. Hang the sinker from the balance beam in the water and counterpoise with lead shot. Place the substance in the right-hand pan, and find its mass by placing weights in the left-hand pan. Now, leaving these weights in the left-hand pan, place the substance in the sinker. The right-hand side is now too light. Remove weights from the left-hand pan to balance, and these weights will obviously give the loss in weight of the substance on being placed in water.

In the case of a substance like wax, great care must be

taken to remove air bubbles from the substance when immersed in water.

RESULTS.—Record all you have done and observed. Give the densities of the substances provided. Check these results by means of tables of densities.

25. Densities by Spring Balance.

APPARATUS.—Spring Balance with Scale, Weights, Beaker, Solids heavier and lighter than water, Liquids.

METHOD.—First stretch the spring slightly by hanging a weight on it, and then by adding weights show that the extension of the spring is proportional to the weight producing the extension.

Then use this property of the spring to find the densities of (1) a solid, which sinks in water, (2) a solid, which floats in water, (3) a liquid.

RESULTS.—Table showing that the extension of the spring is proportional to the weight producing the extension, densities of substances, and liquids provided.

26. Density of a Liquid by the Sinker Method.

APPARATUS.—Delicate Balance, Weights, Sinker, Thread, Beaker, Liquids, Platform over pan of balance.

METHOD.—A suitable sinker is a fairly large glass stopper. Weigh the sinker first in air and then wholly immerse in liquid. The difference in weight gives the weight of a quantity of the liquid which has the same volume as the sinker. Now find this volume by weighing the sinker in water and noting the difference between this weight and the weight in air. Thus find the densities of the given liquids as the ratios of the apparent losses of weight in the liquids and in water.

Note always that no bubbles of air adhere to the sinker when it is immersed.

RESULTS.—Give all your observations and state the densities of the liquids provided.

27. Density of Ice.

Apparatus.—Wide Glass Jar, Narrow Graduated Glass Tube closed at one end, Paraffin Oil, Balance, Weights, Dry Duster, Clean lumps of Ice, Thermometer reading to $-10°$ C., Salt.

Method.—Clean the graduated tube and half fill it with paraffin oil. Next counterpoise the tube and oil on the balance by means of weights or lead shot. A loop of thread will keep the tube upright on the balance. Put powdered ice and a little salt into the glass jar, thus obtaining a temperature a few degrees below 0° C., and introduce the tube containing the paraffin into the jar. When the oil has reached the temperature of the freezing mixture outside, read off its volume on the scale. Dry some small lumps of ice with the duster and slip them immediately into the oil. They will sink, since the density of paraffin oil is less than that of ice. Add in this way a volume of ice of about 30 or 40 c.cs., but be careful that none of the ice rises above the surface of the oil. Read the volume now, and in this way get the volume of the ice which you have put into the oil. Remove the tube from the freezing mixture, dry the outside, and place on the balance pan, and add weights to restore the balance. Obviously the weights added give the weight of the ice. In this way determine the density of ice.

Result.—Density of ice in grams per c.c.

28. Barometer and Pressure Gauge.

Apparatus.—The Mercury Barometer fixed in the Laboratory, Aneroid Barometer, U-Tube, Clip Stand, Indiarubber Tube to connect to gas-pipe, Steel Rule.

Objects.—To read the barometer and a simple pressure gauge.

Exercise 1 (*Barometer*).—The ordinary mercury barometer (Fortin's) has a screw underneath the cistern (which contains the mercury at the bottom of the barometer tube), by means

of which the level of the mercury in the cistern may be adjusted. Turn this screw until the mercury in the cistern just touches the ivory pointer. The tip of this ivory pointer is the zero of the scales by which the height of the mercury in the tube is read. Now turn the screw at the upper part of the tube till the edge of the front slide, the top of the mercury column, and the edge of the back slide are all in line. Read the centimetre scale and vernier and the inches scale and vernier. If in doubt, see Experiment 1 Disturb and re-adjust the mercury and take both sets of readings again. Repeat and tabulate results. Take the mean of the inches readings, convert it to centimetres, and compare the result with the mean of the centimetre reading.

Examine the construction of the aneroid barometer and compare its reading with that of the mercury barometer. Also, if the building be of two or three stories, carry the aneroid barometer to one or two different heights and note the difference in air pressure.

Corrections.—The results obtained as above will be sufficiently accurate for some of the earlier experiments requiring barometer readings. At ordinary heights above sea-level and ordinary temperatures a more accurate value will be obtained by subtracting 0·15 cm. from the observed centimetre reading. The corrections are more fully considered in Part II. on Heat.

Exercise 2 (*Pressure Gauge*).—Find the pressure of the gas in the gas-pipes by putting water into the **U**-tube and connecting one limb of the tube to the gas-pipe. The difference in levels of the water in the two limbs gives the excess of the pressure of the gas in the pipe over the atmospheric pressure. Express this excess in

(1) Inches and centimetres of water.

(2) Millimetres of mercury (taking the density of mercury to be 13·6 gm./c.c.).

(3) Dynes per square cm.

(4) Pounds weight per square inch (taking the weight of a cubic foot of water as 1000 oz.).

Results.—Tables of results of barometer readings. Excess pressure of gas over atmospheric pressure.

29. Boyle's Law (demonstrated).

Apparatus.—Boyle's-Law Tube, Metre Scale, Barometer.

Theory.—Boyle's Law states that "the volume of a given mass of gas is inversely proportional to its pressure provided the temperature is constant."

In symbols this may be written—

$$pv = \text{a constant, for temperature constant,}$$

p and v denoting respectively the pressure and volume of a certain mass of the gas in question.

Exercise.—The given tube is sealed at one end and contains air and a thread of mercury. The tube is mounted in a wooden frame, which can rotate on a screw at its centre. By rotation of the tube, the air between the sealed end of the tube and the mercury thread may be subjected to different pressures. Take five readings of different pressures and the corresponding lengths of air column thus: (1) tube vertical sealed end at bottom—in this case the pressure to which the air is subjected is given by the height of the barometer *plus* length of mercury thread, both being in the same units.

(2) Tube vertical, sealed end at top—here the pressure is given by the *difference* of the above two lengths.

(3) Tube horizontal—here the pressure is that given by the barometer reading alone.

(4) and (5) Tube inclined to the vertical, sealed end down and sealed end up—here the pressures are given by the barometer reading plus or minus the *vertical* height of the mercury thread.

Results.—Tabulate results in columns headed (1) Pressure of gas in cms. of mercury. (2) Length of air column, (this is proportional to volume of air). (3) Product of (1) and (2). The values in the last column should be very nearly the same, thus confirming Boyle's Law. If they differ considerably find

their mean and find what ratio the greatest deviation from it bears to the mean.

30. Boyle's Law (verified).

Apparatus.—Standard Apparatus for Boyle's Law, Barometer.

Preliminary.—The apparatus to be used will be easily understood from Fig. 11.

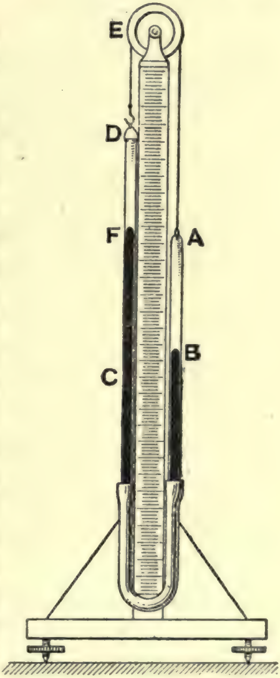

Fig. 11.—Boyle's law apparatus.

The air under observation is contained in the glass tube AB closed at A. The indiarubber tube below contains mercury and connects AB with the glass tube CD, which is open at D. The two tubes are shown connected by a cord passing over a pulley E, and thus by moving one tube up or down the pressure on the air in AB is easily varied. The tubes move up and down in front of a mm. scale. In some forms of the apparatus AB is fixed, and only CD is moved. If care is taken, it is possible to attain a high degree of accuracy with this apparatus. When performing the experiment wait a little after each adjustment to allow the temperature of the air in AB to become the same as the temperature of the air in the room. Also avoid *parallax* when reading the levels of the mercury in the tubes.

Method.—The method is obvious. By raising and lowering CD subject the air in AB to different pressures. Read off the lengths of the air column AB and the corresponding pressures. The lengths of AB are proportional to the volumes.

The pressures are given in mms. of mercury by the barometer reading plus or minus the difference of levels of the mercury in the two tubes. The greatest and least pressures possible to the apparatus should be used and about eight intermediate ones.

TABULATION OF RESULTS.—The following table indicates how results should be set down :—

Reading of air top in closed tube, A.	Reading of mercury top in closed tube, B.	Ditto, in open tube, F.	Atmospheric pressure from barometer.	Volume, v. $\propto$ length, AB.	Pressure, p.	Product, pv.	Deviation from mean.
80·2	100·2	60	76·2	20	116·4	2328	...
...	99·2	52·8	...	19	122·6	2330	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
80·2	...	...	76·1	...	...	...	...
					Mean		

Readings for column (4) must be taken at beginning and end of the observations in case any change in the atmospheric pressure has taken place.

The same applies to column (1) if A is clamped, but if AB moves up and down by the cord the point A must, of course, be read after each shift.

ACCURACY OF RESULTS.—It is not to be expected that the products will all be alike if they are obtained by ordinary multiplication merely. Indeed, it is not legitimate to retain all the figures. For example, if a certain pressure and volume were each expressed by the number 10·4, that would mean that their values were measured correct to about 1 in 100. But if the product were written 108·16, that would imply that its value were certified correct to 1 in 10,000. Whereas it is not known with any more accuracy than its factors, *i.e.* 1 in 100. It should accordingly be written 108 simply.

After obtaining the products with due regard to the above

principle, take their arithmetic mean; find also the greatest deviation of any one product from this mean, and finally express this deviation as a percentage of the mean. This percentage is a gauge of the care and accuracy with which the experiment has been performed.

RESULTS.—Sketch of apparatus. Table of results. Indication of accuracy attained.

31. Impact.

APPARATUS. Impact Apparatus, Blocks of Steel, Brass, Ebonite, etc., Secondary Cell or Daniell Cells.

PRELIMINARY.—If a moving body hit a fixed body with velocity u and rebound with velocity v, the ratio v/u is called the coefficient of restitution, e. The object of the experiment is to determine e between steel and steel, steel and brass, steel and ebonite, etc.

IMPACT APPARATUS.—A convenient form of impact apparatus is shown in Fig. 12.

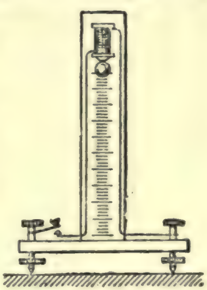

FIG. 12.—Impact apparatus.

On a base fitted with levelling screws stands an upright, which carries a cm. scale along which slides a small electromagnet. At the foot of the scale may be placed blocks of steel, brass, ebonite, etc. The electromagnet may be excited by the current from a secondary cell. On the base is a tapping-key by which the electric circuit may be completed or broken. A steel ball is hung from the electromagnet, when the current is passing, and it is let fall by breaking the circuit.

Where such an apparatus is not available, the ball may be let fall from the finger and thumb at a point near a vertical scale.

METHOD.—Since $u^2 = 2gh_1$, where h_1 is the height through which the ball falls, and $v^2 = 2gh_2$, where h_2 is the height to which it rebounds, $e = v/u = \sqrt{h_2/h_1}$. Therefore, let the ball

drop from various heights h_1, and note the heights h_2 to which it rebounds, and hence calculate e. As it is difficult to read h_2 accurately, several readings must be made for each height, and the mean taken. The ball may be made to rebound accurately up the scale by levelling the base of the instrument.

Find the coefficients of restitution between steel and the various materials provided.

RESULTS.—Tabulate your results in columns headed—materials, h_1, h_2, e, mean e.

32. Coefficient of Friction.

APPARATUS.—Friction Apparatus, Weights, String, Metre Scale.

PRELIMINARY.—Given two bodies A and B (see Fig. 13), having surfaces in contact, one of them being made to slide over the other, if W is the total normal force exerted by A on B, then the force of friction F is known to be nearly proportional to W. The object of the experiment is to show that this is the case, and to determine the coefficient of friction, $\mu = F/W$.

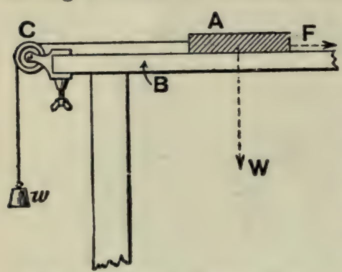

FIG. 13.—Friction apparatus.

METHOD.—Different values are given to W by placing weights on A. Obviously W includes the weight of A. F is determined by means of the weight w, which is attached to A by a string passing over a pulley C. w is adjusted until it is just able to move A over B without acceleration. In this case $w = F$. In an actual experiment, however, it is better to give some value to w and adjust the weights on A. In each determination give A a very gentle start by gently pulling the string.

EXERCISES.—Give w several different values, and find the corresponding values of W. Show by a curve that w is pro-

portional to W, *i.e.* that F is proportional to W, and calculate the value of μ.

If A and B are of wood, find μ when the grain of the wood of A is (i) parallel, (ii) perpendicular to the grain of the wood of B. Then find μ between wood and paper by fastening a sheet of paper on B.

Finally determine μ by taking w and its string away and tilting B until A just slides down. In this case if α is the angle which B makes with the horizontal when A begins to slide down, $\mu = \tan \alpha$. The student must be able to prove this relation between μ and α. If he cannot, he must ask a demonstrator regarding it.

RESULTS.—Tables with columns headed—Surfaces, W, w, μ. Results giving μ for the different pairs of surfaces as determined by the second method.

33. Young's Modulus (by Vernier).

APPARATUS.—Pair of wires about 10 metres long, fixed to a beam, with scale on one and vernier on the other, Weights, say 1 kilogram each, Micrometer Gauge.

OBJECT.—To verify Hooke's Law, which states that the elongation of a wire is proportional to the stretching force, and to find Young's Modulus of Elasticity for the metal of the wire. Young's Modulus is defined as the stretching force per unit area of cross-section of the wire divided by the elongation produced per unit length of the wire, or in symbols,

$$M = F/A \div l/L,$$

where M = Young's Modulus, F = stretching force in dynes, L = length of wire in cms., A = area of cross-section of wire, l = elongation in cms.

Obviously Young's Modulus will be expressed in dynes per sq. cm.

METHOD.—The apparatus consists of two long wires which are hung from the same beam in the roof. One of these is kept taut by a weight. It carries the scale, while the other,

which carries the vernier, has a pan in which weights may be placed.

Exercise 1.—To verify Hooke's Law, make a series of readings of the extensions produced by various weights in the pan both as the weights are added and taken off, and draw a curve to illustrate your results.

Exercise 2.—To obtain Young's Modulus for the metal of the wire insert in the above formula the extreme weight used and the corresponding extension. The cross-section of the wire is obtained from the mean of six readings of the diameter taken with the screw-gauge at places on the wire as far distant from each other as possible.

Results.—Plot a graph connecting loads with elongations and state the value deduced for M. Give the units in which M is expressed. Remember, it may be expressed in other units, *e.g.* pounds weight per square inch. In this case the numerical value for a given substance would be different.

34. Young's Modulus (by Optical Lever).

Apparatus.—Wires about $2\frac{1}{2}$ metres long supported from a bracket on the wall, Weights, Optical Lever, Electric Lamp, Vertical Scale, Metre Scale, Micrometer Gauge.

Method.—Two wires, A and B, hang from a bracket on the wall about 1 inch apart. They carry two metal rings at the bottom. The metal rings are kept parallel to each other by connecting pieces, but are free to move up and down relative to each other. On the inner surface of each ring at the bottom is a piece, which has been ground flat, and on one of these flat pieces are a hole and a slot for two feet of an optical lever. Hang a weight on to each of the wires, A and B, so as to make them hang straight. Place the optical lever so that its front feet are in the hole and slot in the flat piece on A's ring, while its back foot rests on the flat piece inside B's ring. The optical lever has a concave mirror, and by movement of the lamp an image of its filament is obtained

on the vertical scale. Hang weights in succession on one wire, say A, and note the successive deflections on the scale. From the deflection, the distance of the scale from the point of rotation of the optical lever, and the distance between the front feet and back foot of the lever, the extension of the wire for a given weight can be calculated. If in doubt, the student must look up the experiment on the optical lever (experiment 4).

RESULTS.—In this way find Young's Modulus for the metal of the wire.

Repeat with the other wire.

35. Young's Modulus (by Bending).

APPARATUS.—See under Exercise 1 and Exercise 2 below.

PRELIMINARY.—The last two experiments give a method for finding Young's Modulus of metals, which can be obtained in the form of wires. The method of the present experiment is applicable to the determination of Young's Modulus of wood, etc., as well as of metals, in the form of rods, bars, or laths.

EXERCISE 1.—*Rod supported at both ends and bent by loading the centre.*

APPARATUS.—Metal Rods resting on metal knife-edges, Set of Kilogram Weights, Pan for Weights, Telephone, Leclanché-Cell, Electric Micrometer, Metre Scale, Micrometer Gauge.

METHOD.—The rod is supported on two knife-edges, which are placed equally distant from its ends. A scale pan is hung from the centre of the rod, and the rod is bent by putting weights into the pan. The depression of the centre of the rod caused by any given weight is obtained by means of the electric micrometer (see Experiment 2). Let L be the length of rod between the knife-edges, b breadth of rod, d its thickness, l the movement of the centre of the rod when force F is applied at the centre. Then, if M be Young's Modulus,

$$M = FL^3/4bd^3l.$$

Each of the factors being in c.g.s. units, the result is in dynes/sq. cm.

The proof of the above formula may be found in Stewart and Gee's *Practical Physics*, Vol. I., pp. 179–183; or Barton's *Analytical Mechanics*, Art. 464.

MEASUREMENTS.—This determination may be made very exact, if care be taken, as follows. L should be read to the nearest mm. by a metre scale, the knife-edges being set carefully perpendicular to the rod; b is measured in several places with the micrometer gauge, d must be taken with great care at several different places along the bar. If a small error, say 1 per cent., is made in the determination of d, the error in the result is three times as much, viz. 3 per cent., due to the cubing of d. The displacement l should be taken, first as weights are put on, and secondly, as weights are taken off. Two different lengths of rod should be used.

RESULTS.—Tables showing (1) l as F is increased; (2) l as F is decreased. Determinations of Young's Modulus for each metal provided.

EXERCISE 2.—*Rod fixed at one end and bent by loading the other end.*

APPARATUS.—Metal and Wooden Laths, Clamps, Wooden Block, Box of Weights, Metre Scale, Micrometer Gauge, Millimetre Scale mounted on vertical standard, Attachment with pointer for hanging weights at the end of the rod.

METHOD.—One end of the rod is laid on the wooden block and clamped firmly to the bench. The other end of the rod must be so placed that weights may be hung on it. The attachment by which the weights are hung on the rod should have a pointer, which can move up and down a vertical millimetre scale. When weights are hung on, the end of the rod is depressed and the movement is read off on this vertical scale. If L is the length of the rod from the edge of the wooden block to where the weights are hung, then

$$M = 4FL^3/bd^3l.$$

This formula is proved in Stewart and Gee's *Practical Physics*,

Vol. I., pp. 179–182, and in Barton's *Analytical Mechanics*, Art. 464.

Exercise the same care as in 1, and use two different lengths of each rod.

RESULTS.—Tables showing (1) l as F is increased, (2) l as F is decreased. Determination of Young's Modulus for metal and wood.

36. Static Torsion of a Wire.

APPARATUS.—Statical Torsion apparatus, Weights, Micrometer Gauge, Metre Scale.

OBJECT.—To find how the twist of a wire depends upon the twisting couple, the length and the radius of the wire.

PRELIMINARY.—The apparatus has a strong base-piece, shown in the figure which stands on three fairly long legs. This base supports two vertical rods. The top end of the wire under examination is clamped by a cross piece which may occupy any position on the two uprights to suit the length of the wire. The bottom end of the wire supports a heavy disc. Around a groove in the edge of this disc two threads are wound, and the ends of these threads pass over pulleys, and support an aluminium rod with scale pan. The wire is twisted by placing weights in this scale pan. The disc has pointers, which move over a circular scale, and the twist of the wire is read off on this scale.

EXERCISE 1.—To show that $\theta \propto$ G, where θ is the angle of deflection produced by a couple, or torque, G.

Keeping the length of the wire constant, make a series of observations of the angles θ corresponding to different weights, W, in the scale pan. Tabulate results and by means of a curve show that $\theta \propto$ W and so $\theta \propto$ G.

EXERCISE 2.—To show that $\theta \propto l$, where l is the length of the wire that is twisted.

Keeping the weight in the scale-pan constant, make a series of observations of the angles θ corresponding to different lengths, l. Tabulate results and draw curve.

EXERCISE 3.—To show that $\theta \propto 1/r^4$, where r is the radius of the wire.

Keeping the weight in the pan and the length of wire

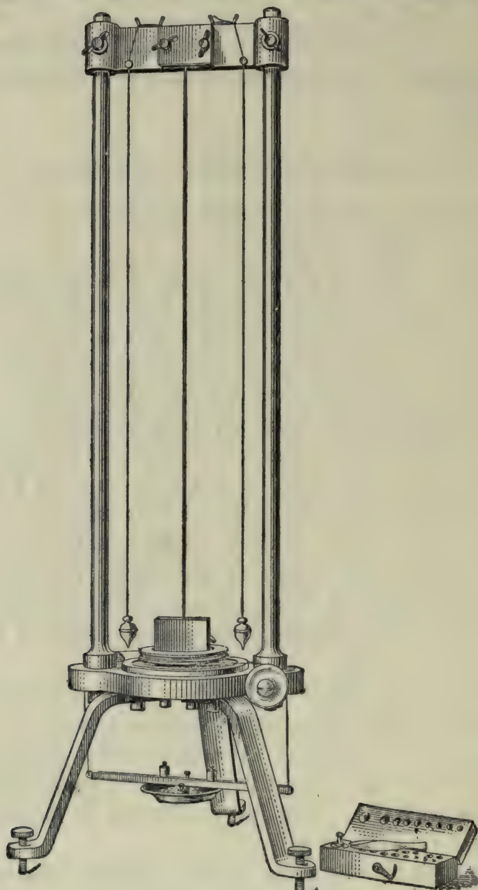

FIG. 14.—Torsion apparatus.

constant, find θ for two or three wires of different diameters, but of the *same material*. Thus show the above is true.

Results.—Collect all your results in tabular form. Plot curves and state the inferences you draw. How does G vary when θ, l, and r all vary?

37. Torsional Oscillations.

Apparatus.—Torsional Pendulums, Weights, Watch with seconds hand.

Preliminary.—The torsional pendulum consists of a heavy wooden disc rigidly fixed at its centre to a wire. The disc is suspended by the wire from a vice on a bracket, the arrangement being such that the length of the pendulum wire may be easily varied. Two such pendulums are provided. They are similar in every respect save that the wires are of different diameters.

Exercise.—Investigate how the period of torsional oscillation of the pendulum depends on—

(1) The amplitude of oscillation.

(2) The length l of wire of pendulum.

(3) The radius r of the wire.

(4) The addition of weights to the oscillating disc.

(5) Change in the distance of the weights from the axis of oscillations.

In making an observation, time at least fifty oscillations of the disc.

Theory shows that the period is independent of the amplitude, is proportional to $\sqrt{l}$, and inversely proportional to r^2. (See Barton's *Analytical Mechanics*, Arts. 262 and 465.)

These three relations the student should verify.

Results.—Tables and curves indicating how the period depends upon the various conditions mentioned. Show whether your results confirm theory or not.

38. Capillary Elevation and Surface Tension.

Apparatus.—Small Dish for Liquids, Capillary Tubes, Clip Stand, Steel mm. Scale, Microscope travelling vertically with scale in eye-piece, Thermometer.

PRELIMINARY.—At the surface of separation of a liquid and a gas, the liquid is in a special state. This specialised portion, usually termed the *skin*, exhibits a constant tendency to contract. It is in a state of tension. It is this state of tension of the surface which gives rise to the elevation or depression of liquids in fine tubes and many other so-called capillary phenomena. The tension varies according to the liquids and gases that are used. In this experiment the elevation of liquids such as water and alcohol in glass tubes is utilised to determine the *surface tension*, as defined below, between water and air, alcohol and air, etc. Liquids like water and alcohol *wet* glass; that is to say, they creep along the surface of the glass (if the glass is clean) where they come in contact with it. And thus if the liquid is contained in a glass vessel or tube, the water-air or alcohol-air surface turns round where the liquid comes in contact with the glass, and finally is parallel to the glass surface.

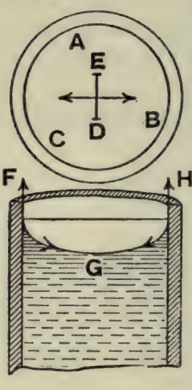

FIG. 15.—Surface tension.

THEORY.—Let ABC be the surface of the liquid in a circular glass vessel. Take a line DE in the surface. Since the surface is in a state of tension, there is a certain *pull* at each side of this line. If we take the line DE as 1 cm. long, this pull is called the *surface tension*, and is generally denoted by T, and is measured in dynes per cm. Where the surface comes in contact with the glass at the side, there will be a similar pull, T, along each cm. of contact of the surface with the glass. Since the liquid with which we are dealing wets glass, this pull will be up and down the glass. For instance, if FGH is the surface of such a liquid in a fine tube, which has been placed in the liquid, there is a pull upwards along the glass, as at F and H, of T dynes along every cm. around the tube. Thus, if the tube is r cms. in radius at this point, there is a total pull upwards of $2\pi r$T dynes. Hence

the liquid rises, and, if the tube is parallel, this force will equal the weight of liquid which has been drawn up into the tube. This, if h is the height to which the liquid has risen, is $\pi r^2 h\rho g$ dynes, where ρ is the density of the liquid and g is the acceleration due to gravity. Thus we have—

$$2\pi r\mathrm{T} = \pi r^2 h\rho g$$
$$\therefore \mathrm{T} = rh\rho g/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A more accurate expression is

$$\mathrm{T} = (h + r/3)r\rho g/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which case allowance is made for the volume of liquid in the meniscus. In what case does $r/3$ become negligible?

If the tube is not parallel it may be shown that equation (1) holds provided r refers to the place to which the liquid rises. (See Barton's *Analytical Mechanics*, Art. 436.)

Method.—In all experiments on surface tension great cleanliness should be observed. Therefore clean the glass tubes and containing vessel thoroughly with caustic soda, nitric acid, and water. If any other liquid than water is to be used, wash out with the liquid all traces of water from the tube and containing vessel. If water is used, do not use distilled water. Water *straight* from the tap, although chemically less pure, has a cleaner surface.

Set the tube vertically in the liquid and see if the liquid rises freely in the tube. If it does not, the tube must be cleaned again. See also that the liquid has wetted the tube thoroughly by moving the tube up and down while its end is in the liquid.

Obtain the height h by means of the travelling microscope (or rule). At the point to which the liquid has risen in the tube, make a scratch with a file and break the tube off at this point. It is at this point that the radius of the tube must be determined. Place the tube horizontally and view the broken end through the microscope, and determine the inside diameter either by means of the micrometer scale in the eye-piece, or by the scale on the microscope. In all probability the bore of the

tube will not be perfectly circular. To correct for this measure two diameters at right angles and take the mean.

The other unknown quantity is the density of the liquid. This may be obtained from tables or by experiment.

EXERCISES.—Find the surface tension between water and air and alcohol and air, etc., using two tubes for each liquid.

RESULTS.—Tables showing all the data required for finding T, and also the values of T for the various liquids.

PART II.—HEAT

39. Thermometry.

Apparatus.—Glass Funnel, Hypsometer, Wooden Clip-stands, Ice, Salt, Indiarubber Bands, Two Thermometers reading from $-20°$ to $105°$ C.

Object.—To find the fixed points of a thermometer, and to compare the readings of two thermometers.

Method.—*Freezing Point.*—Wash the ice so that you may be sure there is no salt about it. Cut the ice into shavings. Fill the funnel with the ice, and, supporting the thermometer with the clip-stand, place the bulb in the ice up to the freezing-point graduation mark. Pack the ice loosely round it and allow the thermometer to stand thus for about ten minutes. Then take the reading and note the error, if any, in the freezing-point graduation. Repeat with another thermometer.

Boiling Point.—Note the construction of the hypsometer. It is so made that the steam in the inner chamber is kept *dry* by the heat from the steam in the outer jacket. Pass a thermometer through the cork in the top of the hypsometer and adjust it so that the boiling-point graduation is just below the cork, and can be raised just above it, when a reading is taken. Boil the water in the hypsometer, and after the steam has been coming off for some time, read the thermometer from time to time until readings taken at intervals of five minutes agree with each other. Note this temperature and read the barometer *immediately*.

From the reading of the barometer obtain the temperature at which the water is boiling from a book of physical tables.

(Ask a demonstrator for the tables.) Before looking out the temperature from the tables reduce the barometer reading to standard conditions by applying the corrections given in the experiment on the barometer, page 35. Note the error in the boiling-point graduation of the thermometer. Repeat with another thermometer.

Comparison of readings of two thermometers.—Bind the two thermometers together with indiarubber bands, and supporting them in a clip stand place them in a beaker of water. Heat up the water and take the readings of the two thermometers at about every 10° as the temperature rises. When a reading is being taken, take the bunsen away, stir the water well, and wait for a minute or two.

A temperature much below 0° C., say, as low as -18 C., may be obtained by pounding a quantity of ice with about half as much salt together in a mortar. Compare the graduations of the two thermometers for temperatures below 0° C.

Tabulate the readings of the two thermometers. Taking one thermometer as a standard, plot a curve connecting the readings of this thermometer with the errors of the other.

RESULTS.—Freezing-point and boiling-point errors of two thermometers. Table and curve showing comparisons of the readings of two thermometers.

40. Linear Expansion.

APPARATUS.—Expansion Apparatus, Two Thermometers, Metre Rule, Heating Arrangements.

OBJECT.—To find the coefficient of linear expansion of a metal.

METHOD.—The expansion apparatus consists of a bar (brass or iron), contained in a long trough to hold water, by means of which the bar may be raised to any desired temperature.

One end of the bar is fixed, abutting against a brass piece

fixed to the bed of the apparatus. On an upright attached to the bar near the other end is a pair of cross-wires, the motion of which is due to the expansion of that portion of the bar from the fixed end to the upright carrying the cross-wires. The motion of the cross-wires is measured by means of a micrometer eye-piece, attached to a support which is fixed in the bed of the apparatus.

The eye lens should first be set so that the cross-wires *of the eye-piece* are clearly seen without any straining of the eye. The object lens should then be set so that an image of the cross-wires *on the bar* is obtained well in coincidence with the cross-wires *of the eye-piece.* It is advisable to get one cross-wire vertical, and to take readings by setting it always at the intersection of the other pair of cross-wires.

Note that in order to avoid error due to the looseness of the screw of the eye-piece, it is necessary always to set the eye-piece in position by turning the screw in the same sense, that is, always by a right-handed motion, or always by a left-handed motion. Thus, if you are trying to set the eye-piece by a right-handed motion of the screw and go a little too far, do not simply go back with a left-handed motion, but go well back and come up again with a right-handed motion.

Find out how the scale on the screw of the eye-piece reads. Heat up the water and take readings every 10° as the temperature rises. Cool the water, and take readings every 10° as the temperature falls. In taking the second set of readings, set the eye-piece by motion of the screw in the same sense as in taking the first set in order to get the readings comparable with each other.

The water must be well stirred. The temperature of the water should be taken at several points along the length of the bar, and, if there is any difference in temperature, the mean of the readings should be taken.

Finally, take the bar out and measure its length from the end that was fixed to the centre of upright carrying the cross-wires. This length should be measured accurately to a millimetre. Plot a curve having as ordinates changes in length

of the bar and as abscissæ temperatures, and calculate the coefficient of linear expansion of the metal of the bar from

$$\alpha = l/L(t_2 - t_1)$$

where α = coefficient of linear expansion, L = length of the bar, t_1 and t_2 = the extreme temperatures, and l = total change in length between the extreme temperatures.

Repeat with a bar of another metal.

Results.—Curves of results. Coefficients of linear expansion.

41. Dilatometer for Liquid Expansions.

Apparatus.—Expansion Apparatus, Thermometer.

Object.—To find the coefficient of expansion of liquids.

Method.—The apparatus consists of a vessel containing water in which are supported two glass bulbs with fine tubes attached. The volume of each bulb and tube up to the zero mark in terms of the internal volume of a centimetre length of the tube (*bulb constant*) is marked on the apparatus, or should be asked for. One bulb contains water, the other contains alcohol.

Note the heights of the liquids in the fine tubes, and, at the same time, the temperature of the water surrounding them. Heat up the water say about 5° C., stir well, and wait until the liquids in the bulbs have attained the temperature of the water outside. Read the heights of the liquids and the temperature. Proceed in this way, raising the water by about 5° C. each time. Repeat these observations when cooling the water 5° C. at a time.

To each reading of a tube add the bulb constant, to obtain a measure of the total volume up to the reading in question.

Tabulate your results as follows :—

Water (50).			Alcohol (42·3).		
Temperature	Scale reading.	Volume.	Temperature.	Scale reading.	Volume.
10·2°	3	53	10·1°	2·1	44·4
15·3°	...	...	15·2°	...	...
19·6°	...	...	20·0°	...	...

The numbers in brackets at the head show the supposed bulb constants for each tube, and must be ascertained.

Plot curves connecting volumes and temperatures. What conclusions can you draw from the forms of the curves as to the coefficients of expansion of the two liquids.

Calculate the mean coefficients of expansion of the two liquids over the given range from

$$\beta = v/V(t_2 - t_1)$$

where β = coefficient of expansion, V = volume of the liquid at the low temperature, t_1 and t_2 = the extreme temperatures of the range for which the coefficient is to be calculated, v = total change in volume between these two temperatures.

Results.—Tables and curves and coefficients of expansion for water and alcohol.

Note.—The coefficient of expansion, β, as calculated from the above observations is not the *real* coefficient of expansion. The bulb containing the liquid expands as the temperature rises, and thus the expansion of the liquid, which is observed, is not the real increase in volume, but is less than that. The coefficient, β, therefore, is less than it ought to be, and it is called the *apparent* coefficient of expansion. It is not hard to show that the *real* coefficient of expansion of the liquid is very approximately obtained by adding the coefficient of volume expansion of the glass of the bulb containing the liquid to the apparent coefficient of expansion of the liquid as obtained above. The coefficient of volume expansion of glass may be taken as 0·000025 per 1° C.

42. Liquid Expansion by Balance Methods.

Apparatus.—Balance and Weights, Thermometer, Specific-Gravity Bottle, Beaker, Glass stopper to act as sinker.

Object.—To find the coefficient of expansion of a liquid.

Theory.—Instead of the equation for expansion given in the last experiment we may write—

$$V_2 = V_1[1 + \beta(t_2 - t_1)]$$

where V_1 and V_2 are the volumes of the liquid at the temperatures t_1 and t_2 respectively.

Since the densities are inversely proportional to the volumes, the mass remaining the same, we may put the above equation in the form—

$$D_1 = D_2[1 + \beta(t_2 - t_1)]$$

where D_1 and D_2 are the densities at the temperatures t_1 and t_2 respectively.

Thus to determine β, we have to find the densities of the liquid at two different temperatures, and to do this we may use any of the ordinary methods.

Method 1. *By Specific-Gravity Bottle.*—If M_1 be the mass of liquid which just fills the specific-gravity bottle at temperature t_1, and M_2 the mass which does so at temperature t_2, and *if we assume that the volume of the bottle is the same at both temperatures*, we may, since densities are proportional to masses if the volume remains the same, write the above equation—

$$M_1 = M_2[1 + \beta(t_2 - t_1)]$$

or

$$\beta = (M_1 - M_2)/M_2(t_2 - t_1)$$

Thus β may be found by determining M_1 and M_2. Clean and dry the specific-gravity bottle and weigh it. In the mean time boil some water for some time to expel the air it contains. Cool this down to the temperature of the air and fill the specific-gravity bottle with it. Place the specific-gravity bottle in a beaker of water, which is some degrees above

the temperature of the air. Leave the bottle there for five or ten minutes to ensure that the water inside the bottle is at the same temperature as the water outside. Note the temperature of the water, take the bottle out, dry it and weigh it. The increase in weight gives M_1.

Without removing the stopper, replace the bottle in the water in the beaker and heat up the water to any desired temperature, say 50°. Keep the temperature constant for from five to ten minutes, stirring all the time. Note the temperature, remove the bottle, dry it and weigh, and thus obtain M_2.

Obtain thus the mean coefficient of apparent expansion of water between, say, 25° and 50° C. Then proceed in the same way to find the mean coefficient between, say, 50° and 80° C.

METHOD 2. *By Sinker.*—Weigh the sinker in air and then in the liquid at two temperatures, say, 25° and 50° C., then, *assuming that the volume of the sinker is constant throughout*, M_1 and M_2 will be given by the apparent losses of weight of the sinker in the water at the two temperatures.

Obtain thus the mean coefficient of apparent expansion of water between, say, 25° and 50° C. Then proceed in the same way to find the mean coefficient between, say, 50° and 80° C.

RESULTS.—Coefficients of apparent expansion of water between the temperatures stated.

NOTE.—Owing to the expansion of the bottle and the sinker only the apparent expansion is obtained. It is not hard to show that in each case the real coefficient is obtained by adding, as in the previous experiment, the coefficient of cubical expansion of glass (if the sinker is made of glass).

If the student cannot show this, he should consult a demonstrator.

43. Barometer Corrections.

APPARATUS.—Fortin's Barometer (see Experiment 28).

The standard atmosphere is defined as that which exerts a pressure equal to the weight of a column of mercury 76 cm

high at the sea-level and at latitude 45°, the mercury being at 0° C. In order to make all barometer readings comparable they must be reduced to standard conditions as follows :—

I. *Zero Error and Capillarity.*—The scale may not be accurate, and in particular the end of the ivory pointer may not be the *true zero* of the scale.

Further, since in a narrow tube the *surface tension* will tend to depress the surface of the mercury, a correction must be applied for the error due to this.

These two corrections are best determined by comparison with a standard barometer, and a certificate giving the corrections obtained by such a comparison is generally provided with a good barometer.

II. *Expansion.*—The *length of the scale* at $t°$ C. is greater than its length at 0° C. Also the *density of mercury* at $t°$ C. is less than at 0° C. ; hence a column of mercury of height h at $t°$ C. is only equal in weight to a column of mercury of height $(h - a)$ at 0° C., a being a correction proportional to the temperature. These two corrections will partly neutralise each other.

If δ be the coefficient of volume expansion of mercury and α the coefficient of linear expansion of brass (the scale being made of brass) it can be shown that, where h denotes the observed and H the standardized height,

$$\mathrm{H} = h[1 - (\delta - \alpha)t].$$

Now $\delta = 0{\cdot}000182$ and $\alpha = 0{\cdot}000020$, so that

$$\mathrm{H} = h(1 - 0{\cdot}000162t)$$

or the correction to the reading h

$$= \mathrm{H} - h = -h(0{\cdot}000162)t.$$

The temperature is obtained from the thermometer attached to the barometer.

III. *Altitude and Latitude.*—Since the weight of a given mass of any substance is different at different places on the earth's surface, being less at any height above the sea-level

than *at the sea-level itself*, and greater at the poles than at the equator, it is evident that the height of the mercury column will depend upon height above sea-level and upon latitude.

It can be shown that

$$H_0 = H(1 - 0{\cdot}0026 \cos 2\lambda - 0{\cdot}00000006k)$$

where H_0 = the standardized height, H = the height as corrected above, λ = latitude, k = height above sea-level in *feet*. (The value of k should be found from a demonstrator.)

IV. *Vapour Pressure.*—Lastly, the space above the mercury in the tube is not a true vacuum, but *mercury vapour* is present, which exerts a downward pressure on the mercury column. This pressure may be taken as equal to $0{\cdot}002t$ mms. of mercury, where t is the temperature in degrees Centigrade. This correction must be added.

RESULT.—Barometer reading corrected as above.

NOTE.—Correction II. is generally much more important than corrections I., III., or IV.; usually it is about -2 mm., all the other three together often amounting to about $+ 0{\cdot}5$ mm.

44. Expansion of Air.

APPARATUS.—Air Expansion Apparatus, Thermometer.

PRELIMINARY.—The volume V_t of a given mass of gas at $t°$ C. is connected with its volume V_0 at 0° C. by an expression of the form

$$V_t = V_0(1 + \gamma t),$$

provided the pressure is kept constant; γ is called the *coefficient of expansion* (or *coefficient of increase of volume*) *at constant pressure.* It is very nearly the same for all gases.

Note that V_0 is the volume at 0° C. In the corresponding expressions in the case of linear and liquid expansion, mention was made only of a low temperature and a higher. To be strictly accurate, the low temperature there should also be 0° C. But the coefficient of expansion is so small that there

we may neglect terms in which the square of the coefficient occurs, whereas here we may not. If the student cannot see how terms involving the square of the coefficient come into consideration, he should consult a demonstrator.

OBJECT.—To determine γ.

EXPANSION APPARATUS.—A convenient form of air expansion apparatus is shown in the diagram, Fig. 16.

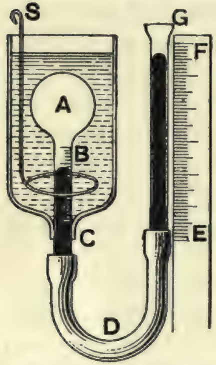

FIG. 16.—Air expansion apparatus.

A is a glass bulb containing pure dry air, BC is a graduated glass tube, CDE is a thick-walled indiarubber tube connected to a glass tube EF. The tube BDF is filled with mercury. EFG may be raised or lowered so that the pressure on the air in A may be adjusted. There is a scale at EF by means of which the height of the mercury in EF above that in BC may be read off. The bulb A is surrounded by a glass vessel containing water, which is used to heat or cool the air in A and in the tube BC. The water may be conveniently heated by passing steam through a brass tube inserted into the water. S is a stirrer.

The volume of the bulb A down to the zero mark at B in terms of the volume of 1 cm. length of the tube BC must be known. (If not marked on, ask a demonstrator for this bulb constant.)

METHOD.—Surround the bulb A with cold water. Wait until everything is at a uniform temperature and adjust the mercury tube EF until the mercury in tube BC stands at the zero mark at B, then the volume of air is given by the bulb constant simply. Take the temperature of the water and read off the height of the mercury in EF above that in BC.

Heat the water through, say, 10° C., stir well and adjust the mercury tube EF until the mercury in EF stands *at the same height above that in* BC *as before.* Obviously the pressure to

which the air in A is subjected is the same as before, if this adjustment is made, and so the condition of *constant pressure* is fulfilled. Read the height at which the mercury in BC now stands, and by adding this reading to the bulb constant, find the new volume which the air occupies at the higher temperature.

Proceed in this way, raising the temperature about 10° C. at a time and adjusting the mercury *each time* for constant pressure (and state in your record that this is done). Make a series of similar observations, cooling the water 10° C. at a time. The hot water may here be siphoned out and a like volume of cold water poured in.

Tabulate your results as follows :—

Temperature rising.				Temperature falling.			
Pressure.	Temperature.	Scale reading.	Volume.	Pressure.	Temperature.	Scale reading.	Volume.
Constant	10°	...	...	Constant	90°	...	...
Do.	15°	...	...	Do.	80°	...	...

Plot a curve connecting temperature and volume, and from the curve find the volume of the air, V_0, at 0° C.

Having obtained V_0, calculate γ by means of the volume at the highest temperature that is reached in the course of the experiment.

Results.—Table and curve of volumes and temperatures. Determination of γ.

Exercise.—Given that the coefficient of expansion of air at constant pressure is 0·00367 per 1° C., find the volume of the bulb down to the zero mark in terms of the volume of 1 cm. length of the tube BC.

45. Increase of Pressure of Air.

Apparatus.—Air Expansion Apparatus, Thermometer.

Preliminary.—The pressure P_t of a given mass of gas at $t°$ C., is connected with its pressure P_0 at 0° C. by an expression of the form

$$P_t = P_0(1 + \delta t),$$

provided the volume is kept *constant.* The quantity δ is called the *coefficient of increase of pressure at constant volume.* It is very nearly the same for all gases.

Object.—To determine δ.

Method.—The apparatus is the same as that described in the last experiment.

Before starting the experiment read the barometer.

Surround the bulb A with cold water, wait until everything is at a uniform temperature, and adjust the mercury tube EF until the mercury in BC stands at the zero mark. Take the temperature of the water and note the height of the mercury in EF above that in BC. Obviously the barometer, *plus* or *minus* the difference in the levels of the mercury in EF and BC, gives the pressure of the air in the bulb A.

Heat the water up say through 10° C., stir well and adjust the mercury tube EF until the mercury in BC stands again *at the zero mark.* Obviously the volume of the air in A is now the same as before, but the pressure which it now sustains is greater. Read the height of the mercury in EF above that in AB, and in this way obtain the pressure of the air in A at this new temperature.

Proceed in this way, raising the temperature 10° C. at a time and adjusting the mercury each time to the zero mark, and state in your record that this is done. Make a series of similar observations, cooling the water 10° C. at a time. For this purpose the hot water may be siphoned out and cold water poured in.

Tabulate your results as follows :—

Temperature rising.					Temperature falling.				
Volume.	Temperature.	Barometer reading.	Difference in mercury levels.	Total pressure.	Volume.	Temperature.	Barometer reading.	Difference in mercury levels.	Total pressure.
Bulb constant	20°	...	...	...	Bulb constant	90°	...	...	...
Do.	30°	...	...	...	Do.	80°	...	...	...
Do.	40°	...	...	...	Do.	70°	...	...	...

Plot a curve connecting pressure and temperature, and from the curve find the pressure, P_0, at 0° C.

Having obtained P_0, calculate δ by means of the highest pressure that is reached in the course of the experiment.

RESULTS.—Tables and curves of pressures and temperatures. Determination of δ.

EXERCISE.—Given that the coefficient of increase of pressure of air at constant volume is 0·00367 per 1° C., find the height of the barometer.

46. Specific Heat by Mixture.

APPARATUS.—Calorimeter, Block or Roll of Metal, Steam Heater, Ordinary Thermometer 0° to 100° C., Delicate Thermometer 0° to 40° C., Clip Stand, Balance and Weights. Liquids, such as alcohol or paraffin oil.

OBJECT.—To determine the specific heat of a substance, *i.e.* the number of units of heat required to raise 1 gram of the substance through 1° C.

(*a*) SPECIFIC HEAT OF A SOLID.

THEORY.—In this method a known mass of the substance is heated to a known temperature, and then immersed in a known mass of water at a known temperature contained in

a suitable calorimeter. A delicate thermometer immersed in the water gives the consequent rise of temperature.

Let M be the mass of the substance of which the specific heat is required, T the temperature to which it is heated, and K its specific heat. Let m be the mass of water in the calorimeter, t_1 the initial temperature of the water, and t_2 its final temperature. Let m_1 be the mass of the calorimeter, and s_1 its specific heat.

The quantity of heat given out by the substance in cooling from T to $t_2 = \mathrm{MK}(\mathrm{T} - t_2)$ calories, and the quantity of heat absorbed (1) by the water in the calorimeter $= m(t_2 - t_1)$; (2) by the calorimeter $= m_1 s_1(t_2 - t_1)$ calories.

Hence, the loss of heat by radiation and the heat absorbed by the thermometer and stirrer being neglected, we have

$$\begin{aligned}\mathrm{MK}(\mathrm{T} - t_2) &= m(t_2 - t_1) + m_1 s_1(t_2 - t_1)\\ &= (m + m_1 s_1)(t_2 - t_1).\end{aligned}$$

Method.—A convenient substance is a metal, preferably in the form of a roll.

Find the mass of the piece of metal, M, and suspend the metal by a thread in the interior of the steam jacket. This should be done as soon as possible, as it takes some time to heat the metal to a suitably high temperature. Insert a thermometer through the cork in the top of the steam heater, and fix it so that its bulb is inside the roll of metal and so that only that part of the scale above 90° protrudes above the cork.

Weigh the calorimeter. Fill the calorimeter about two-thirds full of water and weigh again. Thus obtain m and m_1. The specific heat s_1 depends on the material of which the calorimeter is made. If it is made of iron, $s_1 = 0{\cdot}114$, if of copper, $s_1 = 0{\cdot}094$. Place a delicate thermometer in the water in the calorimeter and support by a clip stand. A small glass rod should also be placed in the water for stirring purposes.

Regulate the supply of steam in the heater so that a certain high temperature, say 90° C., is maintained for at

least five minutes. Then read the temperature, t_1, of the water in the calorimeter accurately to a small fraction of a degree, estimating to tenths of a scale division. Place the calorimeter under the steam heater and drop the roll of metal as quickly as possible into the water. Remove the calorimeter, stir well and note the highest temperature reached, t_2, as accurately as possible.

From these data determine K.

RESULTS.—Two determinations of K, (1) when the highest temperature T is about 80° ; (2) when T is about 90°.

(*b*) SPECIFIC HEAT OF A LIQUID.

By using a solid substance of *known* specific heat, the student should have no difficulty in determining the specific heat of a liquid by the above method.

47. Radiation.

APPARATUS.—Two thin-walled copper vessels, of exactly the same dimensions, one being polished on the outside, the other blackened. The vessels are suspended from stands by threads. Thermometers, Tripod, Bunsen, and Beaker.

OBJECT.—To show the difference in the rate of heat radiation from different surfaces and to test, as far as possible, the truth of Newton's Law of Cooling, which states that the amount of heat radiated per second from a given surface is proportional to the excess of temperature above the temperature of the surroundings.

METHOD.—Take the temperature of the air in the room. Pour into each vessel exactly the same quantity of hot water, say, at about 90° C. Place the thermometers in the water in the two vessels and note the temperatures every half-minute as the temperatures fall, stirring gently all the time.

RESULTS.—Plot two pairs of curves, one pair for the bright and black surfaces, showing the connection between

F

temperatures and times, and another pair showing the connection between the rates of cooling at particular temperatures and the excess of these temperatures above the temperature of the air surrounding the vessels. Do these latter curves substantiate Newton's Law of Cooling or not? If not, can you suggest any reason for discrepancy?

48. Specific Heat of Liquids by Cooling.

APPARATUS.—Cooling Apparatus. Delicate Thermometer, Balance and Weights, Ice, Liquid, such as Paraffin Oil or Alcohol.

OBJECT.—To find the specific heat of a given liquid.

THEORY.—The quantity of heat radiated from a calorimeter in one second at any particular temperature, θ, will be the same whatever liquid is inside the calorimeter, provided nothing else is changed. In this experiment we note the cooling and find the quantity of heat that is radiated per second at a particular temperature when water is inside the calorimeter, and then the cooling when the liquid, of which we require the specific heat, is in it, and from the fact that the two quantities radiated per second are equal, we can determine the specific heat that is required.

A known mass of water, M_1, at a temperature of about $10°$ above the temperature θ is placed in a calorimeter of mass m_1 and specific heat k. The calorimeter is then placed in the cooling apparatus, and the time is observed in which the water cools from a temperature θ_1 about $5°$ above θ to a temperature θ_2 the same number of degrees below θ. If this time be t_1 seconds, the quantity of heat radiated per second at temperature θ is

$$(M_1 + mk)(\theta_1 - \theta_2)/t_1.$$

The water is then replaced by a known mass M_2 of the liquid (of which the volume is as nearly as possible the same as that of the water), and the time t_2 is observed in which the liquid cools through the same range of temperature, $\theta_1 - \theta_2$.

If s be the unknown specific heat of the liquid, the quantity of heat radiated per second in this case at temperature θ is

$$(M_2 s + mk)(\theta_1 - \theta_2)/t_2.$$

Equating these two quantities of heat, we have $(M_1 + mk)/t_1 = (M_2 s + mk)/t_2$, and thus, knowing M_1, M_2, m, k, t_1 and t_2, we can determine s.

METHOD.—Fill the ice chamber and the cover with lumps of ice, but do not put any ice in the air space surrounding the calorimeter. Weigh the calorimeter. Warm water in a beaker (*not in the calorimeter*) to about 50°. Fill the calorimeter about three-quarters full. Weigh the calorimeter again. Place the calorimeter in the cooling apparatus. Insert the thermometer and cover up. Watch the thermometer and record the temperature every half-minute throughout the range chosen.

Clean out the calorimeter and repeat the operation in every detail with the liquid.

RESULTS.—Plot curves connecting temperatures with times for the liquid and for water. Determine s for the liquid.

49. Latent Heat of Water.

APPARATUS.—Calorimeter, Delicate Thermometer, Balance and Weights, Ice, Duster.

OBJECT.—To determine the latent heat of water, *i.e.* the number of units of heat required to change one gram of ice at 0° C. into water at 0° C.

THEORY.—A known mass of water, M, at a known temperature t_1 is contained in a calorimeter. Some pieces of ice are then dropped in and, when the ice is melted, the temperature t_2 is immediately noted. The calorimeter is then weighed and the increase in weight gives the mass, m, of ice, which has been melted.

If m_1, k, be the mass and specific heat of the calorimeter, the heat given out by the water and calorimeter in cooling

$$= (M + m_1 k_1)(t_1 - t_2)$$

and the heat absorbed by the ice

$$= (m\mathrm{L} + mt_2),$$

where L is the latent heat of water. Thus, provided no heat is lost by radiation and evaporation,

$$(\mathrm{M} + m_1k_1)(t_1 - t_2) = m(\mathrm{L} + t_2).$$

Method.—The student should have no difficulty in seeing how to proceed from the above.

Precautions.—1. The ice must be in rather small pieces, about the size of a nut, so as to allow it to melt quickly, and must also be as dry as possible. We may attain this by breaking the ice into fragments, and putting it piece by piece into the calorimeter, brushing off from each piece as it is put in all traces of moisture with a dry duster folded several times so that the ice cannot receive heat by conduction from the hands.

2. In order to avoid loss or gain of heat by radiation, the temperature of the water used may be raised above that of the room before introducing the ice by warming some in a beaker and pouring it into the calorimeter before weighing, and the quantity of ice taken should be such that the temperature of the water at the end of the experiment may be about as much below that of the room as it was above it initially. A preliminary rough experiment may be made to determine this approximately. The temperature of the water must be noted just before the ice is introduced. It is very important that the ice should be quickly melted, to avoid error due to heat taken up by the calorimeter when cold. So do not let the final temperature be much below 10°.

Result.—The latent heat of water may be taken as 80 calories per gram. The value obtained by the above method ought not to differ from this by more than 1 per cent. Repeat the experiment if necessary.

50. Latent Heat of Steam.

Apparatus.—Steam Producer, Calorimeter, Delicate Thermometer, Balance and Weights, Ice.

Object.—To determine the latent heat of steam; *i.e.* the number of units of heat required to change one gram of water at 100° C. into steam at 100° C.

Theory.—Dry steam is passed into a weighed quantity of water at a known temperature for a short time, and the rise of temperature noted. The contents of the calorimeter are again weighed, and the increase in the weight of water gives the steam that has passed in.

Let M be the mass of water in the calorimeter, m_1 the mass of the calorimeter, s the specific heat of the material of which it is composed, t_1 the inital temperature, t_2 the final temperature after a mass m of steam has been passed in, L the latent heat of steam, T the temperature of the steam entering the calorimeter.

Heat given out by steam $= \mathrm{L}m + m(\mathrm{T} - t_2)$.

Heat absorbed by calorimeter and water

$$= (\mathrm{M} + sm_1)(t_2 - t_1).$$

Hence, neglecting heat lost by radiation, etc.,

$$\mathrm{L}m + m(\mathrm{T} - t_2) = (\mathrm{M} + sm_1)(t_2 - t_1).$$

Precautions.—1. Do not let the final temperature of the water rise much above 25° C., otherwise loss of heat due to radiation, etc., causes error.

2. As in Experiment 49, care was taken to introduce *dry ice* into the calorimeter, so here *dry steam* should be used, as far as possible, *i.e.* steam which does not carry with it particles of water in the *liquid state*.

For this purpose allow the steam to come off briskly for some time before putting the nozzle into the water.

3. Loss or gain of heat by radiation may be avoided by a method similar to that mentioned in Experiment 49. Make the initial temperature, say 10° below the temperature of the

room, by placing the calorimeter in ice, and then pass the steam until the temperature has risen 10° above the temperature of the room.

Result.—The latent heat of steam is very nearly 536 calories per gram. It is more difficult to get a result within 1 per cent. of this than in Experiment 49. Repeat the experiment until a good result is found.

51. Curve of Cooling of an Alloy.

Apparatus.—Two Vessels containing Alloys of different compositions, Thermometer reading to about 350° C., Watch, Tripod, and Bunsen.

Method.—The alloy, say, of lead and tin, is contained in an iron vessel surrounding a tube in which mercury is placed. The bulb of the thermometer dips into this mercury.

Heat the vessel until the alloy is all melted, in the case of a lead and tin alloy, say, to 300° C. Remove the flame and allow the vessel and its contents to cool, reading the temperature of the thermometer every half-minute, until the alloy has completely solidified. Plot a curve, taking times as abscissæ and temperatures as ordinates. If care has been taken, this curve will certainly show *one* temperature, and *if* the alloy is a suitable one, *two* temperatures, at which the rate of cooling is less than at other temperatures. At these particular temperatures a process of solidification is going on and the temperatures indicate the melting points characteristic of the alloy.

Results.—Curve of cooling of an alloy. Melting point or melting points observed.

52. Melting Point of Wax.

Apparatus.—Paraffin Wax, Evaporating Basin, Bunsen, Wooden Clip, Beaker, Glass Tubing.

Methods.—1. Heat some paraffin wax in a porcelain evaporating dish to a temperature of about 90° C. Dip into

this the bulb of a thermometer and allow it to stay for some time in order that the bulb may get thoroughly warm. Withdraw the thermometer, allowing any superfluous wax to drain off; a thin film of the wax will then remain on the bulb. Immerse the bulb in cold water and heat the water gently, watching the bulb carefully.

The film will disappear at a particular temperature; note this temperature; it is probably above the melting point.

Withdraw the flame and allow the bath to cool, noting the temperature at which the film reappears. Repeat the observation, heating and cooling the bath very slowly, until the two readings differ by not more than 0·5°. The mean of the two may be taken as the melting point.

This method is very useful when only a small quantity of the substance can be obtained.

2. Draw out a fine capillary tube of glass; place its end under the melted paraffin and draw up sufficient to fill the tube. Allow this to become solid. Break off a small piece of the tube and fasten it by a thin indiarubber band to the bulb of the thermometer. Place the thermometer in water. Heat the water, watching the paraffin carefully. Note the temperature at which it becomes clear.

Allow it to cool and note the temperature at which it again becomes visible. Repeat the observations, heating and cooling the bath very slowly until the two temperatures do not differ by more than half a degree. Take the mean of the two as the melting point.

Note.—It is essential to use clean wax for these determinations, as otherwise the wax will not become transparent at a definite temperature.

Results.—Obtain the mean of several determinations of the melting point by each method. Compare the mean values obtained by the two methods.

53. Boiling point of a Liquid.

APPARATUS.—Special U-tube, Liquids such as alcohol and ether, Beaker, Thermometer, Mercury.

METHOD.—The special U-tube employed is shown in Figure 17.

Put a little clean mercury into the U-tube. Tilt the tube until the mercury comes up to the stop-cock A. Close the stop-cock. Put some of the liquid in the funnel B. Open the stop-cock and tilt the tube until the mercury stands slightly higher at C than at D. Then close the stop-cock.

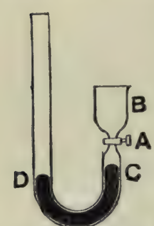

FIG. 17.—Boiling point.

Place the U-tube in a beaker of water and heat the water gently, stirring carefully all the time. At a particular temperature it will be noticed that the mercury at C begins to descend. Remove the bunsen at once, stir well and take the temperature. This is the temperature at which the liquid begins to boil.

Repeat the experiment until you get two or three consistent results.

RESULTS.—Mean determination of the boiling points of, say, two liquids.

54. Hygrometers.

APPARATUS.—Dines' Hygrometer, Daniell's Hygrometer, Regnault's Hygrometer, Wet and Dry Bulb Hygrometer, Thermometers, Ether, Ice, Tables of Vapour Pressures.

OBJECT.—To determine the dew-point and other quantities dependent on the moisture of the atmosphere.

THEORY.—Hygrometers are used to determine the amount of water vapour present in the air.

There are three quantities respecting the water vapour present in the air which are sometimes required, viz. (1) its pressure ; (2) the mass of it in unit volume ; (3) the relative

humidity of the atmosphere. All these can be ascertained, if the *dew-point* has been determined and hygrometers are used for this determination. The *dew-point* is the temperature at which the atmosphere would be saturated by the amount of water vapour actually present at the time of observation.

(1) *Water Vapour Pressure.*—The water vapour in the atmosphere exerts a pressure and is subject to its own pressure, quite independently of the air that is occupying the same space. The pressure which the water vapour must have in order to be *saturated* at any particular temperature is known, and may be obtained from a book of physical tables.

Thus to find the pressure of the water vapour in the room, the dew-point is found, and then from tables the pressure of water vapour required to saturate at this temperature is read off. This must be the pressure of the water vapour actually present in the room.

(2) *Mass of Water Vapour present in* 1 *c.c.*—Having determined the pressure of the water vapour in the room, the mass present in 1 c.c. may be calculated as follows. The density of water vapour at any temperature and pressure under which it can exist is 0·623 of that of air under the same temperature and pressure. But the density of dry air is 0·00129 gm. per c.c. under standard conditions. Thus, at temperature $t°$ C. and pressure p mm. of mercury the mass of water vapour per c.c. is $0{\cdot}00129 \times \frac{273}{273+t} \times \frac{p}{760} \times 0{\cdot}623$.

(3) *Relative Humidity.*—This is the ratio of the quantity of water vapour per c.c. actually present in the room to the quantity per c.c. which would be present if the air were saturated. It is easily seen from (2) above that this ratio is the same as the ratio, pressure of water vapour actually present to the pressure required for saturation at the temperature of the room. Having found, therefore, from the dew-point the pressure of the water vapour in the room, find from the tables the maximum vapour pressure at the temperature of the room and the ratio of the two gives the relative humidity.

EXERCISES. — Find the dew-point by the following

hygrometers. 1 (*Dines'*).—Clean carefully the surface of the blackened glass with a dry duster and rub it very dry. Run ice-cold water through the apparatus, watching carefully the surface of the glass. Directly this is dimmed by moisture condensing upon it read the temperature indicated on the thermometer. This temperature will be slightly below the dew-point. Turn off the water and note the temperature at which the film of moisture begins to disappear. This temperature will be slightly above the dew-point. The mean of the two may be taken as the dew-point. Repeat these observations two or three times.

The hygrometer should be so placed that the glass reflects the light of the sky and accordingly presents a uniform appearance which is at once disturbed by a deposit of dew. Care should be taken to stand as far as possible from the instrument so that there may be little risk of a premature deposit of dew.

2 (*Daniell's*).—This instrument consists of two bulbs communicating with each other by means of a bent tube; one bulb is of blackened glass and has the bulb of a thermometer placed within it; the other is surrounded by a muslin bag. Sufficient ether has been placed within the instrument to fill one of the bulbs.

The instrument must be tilted so as to cause the ether to flow into the blackened bulb. Ether is then poured drop by drop on to the muslin bag; as it evaporates it cools the bulb and the ether from the other bulb will distil over, cooling the blackened bulb by its evaporation. The deposit of dew upon the blackened surface must be observed and the dew-point determined in the same way as in 1 above.

3 (*Regnault's*).—This instrument also depends for its action upon the cooling brought about by the evaporation of ether. The ether is contained in a tube, the upper part of which is of glass, the bottom being made of silver. Into this ether dip a thermometer and a tube communicating with the outside air. The top of the apparatus is closed by a cork through a tube in which air can be drawn from the vessel by

means of an aspirator, its place being supplied by fresh air which bubbles through the ether and causes it to evaporate; the dew-point is found by noting the temperature at which moisture is deposited on the silvered surface.

After the temperatures of deposit and disappearance have been found roughly, make some careful determinations, drawing the air through very slowly. The dew-point can be found to within one or two-tenths of a degree.

If great accuracy is desirable it is well in all cases to read the thermometer and observe the deposit by means of a telescope placed 8 or 10 feet away. Any error due to the body of the observer being too close to the instrument is then rendered negligible.

4 (*Wet and Dry Bulb*).—This instrument, which is largely used in the determination of the hygrometric state of the air for meteorological purposes, consists of two thermometers placed side by side. The bulb of one of these is freely exposed to the air; whilst the other is surrounded by a muslin bag to which is attached a cotton wick which dips into a vessel containing water. Since water will generally be evaporating from the surface of the muslin bag, the thermometer bulb which it surrounds will be at a temperature lower than that of the air.

This difference of temperature depends upon the rate at which water is evaporating from the surface of the bag; this again varying directly as the difference between the amount of vapour present in the air, and the amount which air at that temperature can hold. The rate of evaporation also depends upon the pressure of the atmosphere, diminishing with an increase of pressure.

Tables are provided with the instrument, by means of which the dew-point may be obtained. But the instrument may be standardized as follows by means of another hygrometer.

If t, t' be the temperatures of the wet and dry bulb respectively; h the barometer reading at the time; p' the maximum pressure of water vapour at the temperature of the

air, and p the pressure of the water vapour actually present in the air

$$t' - t \propto \frac{p' - p}{h}$$

or

$$t' - t = A\frac{p' - p}{h}$$

where A is some "constant" which depends upon the construction and surroundings of the instrument.

t', t, h are found by observation, p' from tables, and p from another hygrometer, and thus A is determined. And, A having been determined, the apparatus may be used thereafter for the determination of p, and so of the dew-point.

Find the constant A for this particular apparatus.

Results.—Determination of the dew-point by each of the four hygrometers. Determination of A. State the pressure of the water vapour in the air. Find the amount of vapour present in 1 litre of the atmosphere, and also the relative humidity of the atmosphere.

PART III.—LIGHT

55. Photometry.

APPARATUS.—Two Sources of Light, Rumford's, Joly's, and Pierced-Card Photometers, Metre Rule.

OBJECT.—The object of this experiment is to compare by three methods the intensities, or candle-powers, of two sources of light.

THEORY.—The intensity of a source of light may be estimated by the illumination produced by it when falling normally on a given surface at a given distance. But since at double the distance a fourfold area is included in the same cone of rays diverging from the source, clearly the illumination produced by any source varies *inversely* as the square of its distance from the surface illuminated. Thus, if two different sources produce equal illuminations at different distances, d_1 and d_2, their intensities, S_1 and S_2, are *directly* as the squares of these distances, or

$$\frac{S_1}{S_2} = \frac{d_1^2}{d_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is the principle underlying all photometric methods. The equality of illuminations simultaneously produced by the two sources must be judged by the eye under the special arrangements presented for convenience by each form of photometer. The two distances are then measured and the ratio of intensities found from equation (1).

METHOD.—1. *Rumford's Photometer.*—In this arrangement the shadow of a vertical rod is cast upon a screen by one source of light. The second source is then placed so that a

second shadow is seen side by side close to the first. The rod should be about three inches from the screen and the sources several feet away. They must be adjusted until the two shadows appear of equal illumination. The distances to be dealt with are clearly those from the *screen* to each of the sources.

2. *Joly's Photometer.*—Here the sensitive part consists of two blocks of paraffin wax bound together with a sheet of tin foil between them. This is placed between the two sources of light and adjusted until the diffused illuminations in the two blocks of wax appear equal. The distances to be brought into account are clearly those from the tin foil to each source.

3. *Pierced-Card Photometer.*—Here, too, the sensitive part is placed between the sources. It consists of a piece of card bent at a right angle, the part next the observer being pierced. Suppose this front part to be illuminated by the source on the right, then through the opening is seen the back part which is illuminated by the source on the left. These two parts must receive the light at *equal angles of incidence.* The sources are then adjusted till the opening pierced in the front half of the card almost disappears. The distances are then from the opening to each source.

RESULTS.—Test the sources of light provided by each of these methods and tabulate your results. Note also that if the sources are not symmetrical they should be tested with various sides turned towards the sensitive apparatus. For example, a wax candle well snuffed or a good incandescent gas mantle send practically the same quantity of light in every horizontal direction, but an incandescent electric lamp sends rather different quantities in different directions, so must be tested on all sides.

56. Plane Mirror.

APPARATUS.—Plane Mirror and Two Knitting Needles each placed vertically on stands, Drawing Board and Paper,

Ordinary and Drawing Pins, Protractor and Steel Rule (Sextant or Anglemeter).

Objects.—The objects of this experiment are to illustrate and render familiar (1) the laws of reflection, (2) the construction for the image formed by a plane mirror, and (3) the use of the sextant.

1. Laws of Reflection.—Draw a straight line on the paper and place the mirror so that its silvered back coincides with this line. Stick two ordinary pins vertically into the board about 12 cms. apart so that the line joining them shall meet the mirror obliquely. Look into the mirror with one eye closed and move your head till the images of the pins come into line. While in this position stick two more pins vertically about 12 cms. apart and so as to be in line with the images of the other two. Remove the mirror and the pins and draw two straight lines, one through the positions of the first pair of pins and another for the second pair. These lines should meet on that representing the back of the mirror.

Measure the angles between the mirror-line and each of the pin lines. These lines should be found equal; if so, the law of reflection is confirmed.

Repeat with other positions of pins.

2. Position of Image.—Put the mirror to its line again and place one knitting needle in front of it and one behind. Adjust the needles until in all positions in which the image of the front needle is visible the back needle appears to be a continuation of it. To find this position, move your head from side to side, and if the back needle appears to follow you bring it nearer, and *vice versâ*. When the adjustment is complete measure the distance of each needle from the silvered back of the mirror; these should be equal. Note also the line of the mirror and that joining the positions of the two needles; these should be at right angles. If so, the rule is confirmed that the image in a plane mirror is as far behind the mirror as the object is in front and on the normal to the mirror through that object.

Make another test in like manner.

3. If a sextant or anglemeter is available, use it to find the angle subtended at the eye by two distant objects. The action and use of the instrument depends upon plane reflection, and may be gathered from inspection or by consulting a demonstrator.

RESULTS.—Show by diagrams and brief description all you have done, and state your conclusions.

57. Concave Mirror.

APPARATUS.—Concave Mirror and Holder, Large and Small Pins, Corks, Metre Rule, Small White Card.

OBJECT.—The object of this experiment is (1) to find the focal length of a concave mirror, and (2) to observe the sizes of the images produced by it under various conditions.

THEORY.—If a small object is placed on the axis of a mirror its image will be on the axis also. Let all distances be measured from the mirror, and if against the incident light be considered positive, if with it negative. Thus the radius of curvature of a concave mirror is positive, as in measuring it from the mirror we pass against the incident light. Hence its focal length, being half this radius, is positive also. The radius, focal length, distances of object and image will be denoted respectively by r, f, u, and v. Then we have the following relation for mirrors.

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, if a small object standing at right angles to the axis of the mirror have length q and its image have length p, we have the following relation for mirrors.

$$\frac{p}{q}=-\frac{v}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

See Fig. 18, in which the capital letters indicate the

points whose distances are given above by the corresponding small letters.

In using these formulæ the values of the quantities experimentally obtained must be introduced with the correct algebraic sign and then the equation solved for the unknown sought. This will then be found correctly both as to numerical value and algebraic sign. Thus in solving (1) for v, if it is found to be -10 cm., that means the image is 10 cm. *behind* the mirror; or, if from (2) we find $p/q = -2$, that means that the image is twice the length of the object but *inverted*.

The algebraic signs in equations (1) and (2) must not be

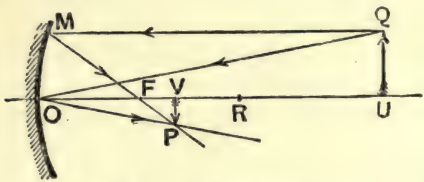

FIG. 18.—Concave mirror.

tampered with in anticipation of any unknown quantity sought being probably negative.

GRAPHICAL RELATION.—Equation (1) may be exhibited graphically as follows. Let axes at right angles be taken and plot a point P whose co-ordinates are $(+f, +f)$. Draw any straight line through this point and produce it to meet the axes. Then it may be shown that its intercepts on the axes of x and y are respectively u and v. Thus one pair of values u and v gives one position of the line, a second pair gives a second position. And the focal length is given as the co-ordinates of their intersection without the necessity for calculating any reciprocals.

FOCAL LENGTH BY SIMPLE METHODS.—1. It is seen by equation (1) or by the graphical relation, that when $u = \infty$, $v = f$. Hence we have the simplest possible method for obtaining f, if a distant source of light is available. For a

mirror whose focal length is only a few inches, an object from 50 to 100 feet away may be taken as giving practically parallel rays as though from infinity. Thus taking a distant window in a long room or a vane, chimney or finial of some outside building sharply relieved against the sky as the object, and using the card as a screen, you obtain the distance between mirror and screen for a sharp image. This distance is the focal length required.

2. Referring again to the relation between u, v, and f, we easily see that when u and v are equal they are each $2f$. This furnishes another very simple method for finding f. Use now for the object a pin, or a piece of white card, fixed in a cork, the mirror being in a vertical plane facing the pin. With one eye closed look over the pin head at the mirror and adjust the pin till its inverted image appears standing on it. The exactness of this adjustment is tested by moving the head from side to side, when, if the image seems to move the same way (relatively to the pin), the pin needs moving nearer the mirror, and *vice versâ*. When the right position is obtained $u = v = 2f$. This result should agree with that found by the first method.

3. Size and Position of Image.—In the position where object and image coincide, by placing a second pin at the side of the first and about 2 cm. from it a second image will be formed. By moving the cork sideways these images may be arranged to appear standing on the pins one on each. This shows that the image is the same size as the object and thus affords an illustration of equation (2) for $u = v$. Now use as object two pins about 4 cm. apart and further from the mirror. Place a cork with one pin in and adjust till one of images appears to stand on this pin. Then place a second pin in the cork at the side of the first so that the two images appear to stand on the two pins. Then by measuring v, u, p, and q, you should get another confirmation of equation (2). Moreover, these values of u and v should furnish another determination of f by equation (1). Repeat in other positions. Note that both equations still apply if

v becomes negative, *i.e.* the image is erect behind the mirror.

RESULTS.—Summarize your results in a little table, also plot a diagram and find the focal length by it.

Can the image ever be greater than the object?

Make ray diagrams to illustrate any interesting positions obtained. In this and any other ray diagrams for mirrors (or lenses) it is generally sufficient to draw from the end of the object two rays, one to the centre of the mirror (or lens), and one parallel to the *axis* of the diagram (which is the normal at the centre of the mirror or lens). This practice is illustrated by the rays QO and QM in Fig. 18.

58. Convex Mirror.

APPARATUS.—Convex Mirror and Holder, Large Pins, Corks, Metre Rule, White Card.

OBJECTS.—(1) To find the focal length of a convex mirror, and (2) to observe the size of the images produced by it under various conditions.

THEORY.—The theory given for the concave mirror is valid for the convex mirror also.

FOCAL LENGTH.—Place the mirror facing you, and in front of it mount on a cork a pin or strip of card so high that its image reaches to the top of the mirror. Now place a second equally tall pin or card on a cork behind the mirror, and adjust it to coincidence with the image of the first as found by moving the head from side to side. Remember that when one of two things (or images) seems to follow the head's motion, that one is the farther one of the two.

Measure the distance u from the mirror to the front pin, and the distance v from the mirror to the back pin. Call the latter distance negative, since it is behind the mirror and so in the direction of the incident light. Put these values of u and v in Equation (1) of Experiment 57 and calculate f, the focal length. This quantity will come negative, but the

student is warned that he *must not tamper* with Equation (1) in anticipation of f being negative. Thus, if $u = +30$ cm., and $v = -10$ cm., we find by (1) that $f = -15$ cm., and r, the radius of curvature, $= -30$ cm.

Other positions of object and image are now to be found, the values of u and v measured, and those of f calculated.

The graphical method can also be used with the v's now taken downwards, because they are negative. The f will then be shown as a negative quantity.

Images.—Equation (2) of Experiment 57 may be confirmed as in the case of the concave mirror. It should be noticed that the image is always erect for the convex mirror, thus p and q should be of the same sign. But they always are so, since u and v are always of opposite signs.

Results.—Give sketches of all you have done, with ray diagrams for some cases; draw up tables of observations and focal lengths calculated from them. Can you ever obtain with a convex mirror a real image by the light from an actual object?

59. Refraction through Cube.

Apparatus.—Cube or Parallel Block of Glass, Pins, Set Square, Rule, Protractor, Paper.

Objects.—(1) To test the law of refraction and find the index of refraction for the glass supplied, and (2) to locate the images formed by refraction at a plane surface.

(1) Law of Refraction.—Place the block of glass on the paper, pencil a trace round it, and keep the block to this position until the pins are adjusted. Thrust one pin vertically in the board just behind the block, a second some distance away, say 12 cm., the line joining the pins making an angle of about 45° with the face of the glass block. With one eye look through the block at these pins, and place two others vertically on this side so as to appear in a line with the other two. Remove the block and pins, draw a line through the positions of the two back pins, another through those of the

two front ones, also join the intersections of these lines with those for the two faces of the glass block. The three lines just drawn represent the path of the rays outside and inside the glass, and if all is correct the two paths in air are parallel.

Draw normals to the refracting faces where the paths of the rays cross, producing the normals each way. Measure with the protractor the angles between the rays and the normals, *i.e.* the external angles of *incidence*, i, at both faces of the block, and the internal angles of *refraction*, r. Next take the ratio of sin i to sin r. Or, with the steel rule the *sines* of these angles may be found directly without measuring the angles themselves.

Repeat the operation with all the angles different, each time taking the ratio of the sines. All the values so obtained of this ratio should be equal or nearly equal to each other.

If practically equal, the law of refraction is confirmed and the mean value of the ratio is your determination of the index of refraction for the glass under test. It is often denoted by μ, and may be represented by the equation

$$\mu = \frac{\sin i}{\sin r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(2) Images by Plane Refraction.—Place the block on the paper and pencil a trace round it. Next, *either* stick a pin vertically *close up* to the back face and near one side of that face, *or* obtain a vertical mark near one side of the back face by sticking a strip of gum paper on it. The position of pin or paper is indicated by Q in Fig. 19.

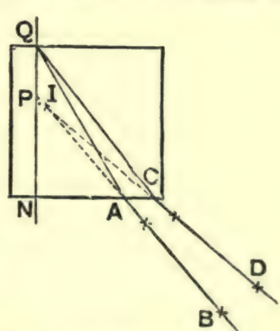

Fig. 19.—Images by plane refraction.

Then, on looking through the block at the line Q, it will appear to be nearer than it is, and the more so if it be looked at very obliquely; in the latter case it appears to move to one side also. To locate the image, stick a

pin vertically near the front face and another at a distance from the face, so that on looking with one eye both these pins seem in a line with the back line at Q. Though these three things appear in line, the light has been refracted on leaving the glass, the path being as shown by QAB. Another pin may now be stuck in close to the front face, and a corresponding position away from the face, as shown by the lines QCD. Then, evidently, if we were to produce the rays BA and DC back, as shown in dotted lines, they would meet in I, which is the image of Q by light emerging through the part of the face AC. Let any such line, BA, be produced to meet the normal NQ in P; then it may be seen that the index of refraction of the glass is given by

$$\mu = \frac{AQ}{AP} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for sin i = AN $\div$ AP, and sin r = AN $\div$ AQ. A number of lines like AB and CD should be determined, the block removed and a number of intersections like I found: the locus of I is called a *caustic*. If A approached very close to N, P would move a little nearer to Q. In that case we should have

$$\mu = \frac{NQ}{NP} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

RESULTS.—Give careful sketches and descriptions of all you have done and observed, tabulating neatly your observations and conclusions.

60. Refractive Index by Microscope.

APPARATUS.—Reading Microscope with Vertical Scale and Vernier, Glass Cubes or Blocks, Liquids, Small Beaker with Mark on inside of bottom, Lycopodium (or Cork Filings).

OBJECT.—To obtain the refractive indices of the various glasses and liquids provided.

METHOD.—The principle to be followed is that of the second part of Experiment 59, but the present equipment

allows more refinement in working, and so permits more accurate results.

Focus on the stage and denote by a the reading on the vertical scale of the microscope. Place a block of glass in position and focus upon its upper face, let this reading be called b. Finally focus upon the stage seen *through* the glass, and let this reading be denoted by c. Then the true thickness of the glass is $(a - b)$, and the apparent thickness is $(c - b)$. Hence, for the refractive index, we have

$$\mu = \frac{a - b}{c - b}$$

The same principle is carried out with the liquids, the mark inside the glass being focussed upon without and with the liquid for the readings a and c respectively. For the reading b, the upper surface of the liquid is located by sprinkling upon it a few lycopodium seeds. If these seeds sink in any of the liquids provided, use the cork filings instead.

Results.—Sketch the arrangement in use, describe what you have done, and tabulate all your observations and calculated values for the refractive indices.

61. Refraction through Prism.

Apparatus.—Prisms, Pins, Protractor, Drawing Board, and Paper.

Object.—The object of this experiment is to note the angular deviation of the light produced by a prism, and thence to deduce the index of refraction of the glass.

Method.—Place the prism on the paper and stick two pins, PQ, vertically on the far side of it and about 12 cm. apart, as shown in Fig. 20. Look through the prism with one eye closed and rotate it slowly in one direction about a vertical axis. It will be seen that during this rotation the two pins first seem to move past one another, then pause, and finally reverse their relative motions. Leave the prism in the position

corresponding to the pause in the pins' motion and make a pencil trace round it. This is the position of *minimum* deviation and has important properties. Now place two other pins, RS, vertically in the front of the prism so as to appear in a line with the back two. Remove the prism and the four pins, draw a line through the back two, PQ, and produce it

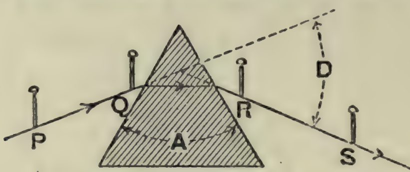

FIG. 20.—Refraction through prism.

forwards, also a line through the first two, RS, and produce it backwards. Measure the acute angle D between these two lines: it is the angular deviation produced by the prism in question. Measure also the angle A between the refracting faces of the prism used in the experiment. Then it is shown in the theory of optics that the index of refraction μ is given by

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}$$

Calculate the value of μ from this equation. Repeat the experiment with a second prism.

RESULTS.—Describe with sketches the method followed and give full details of the observations made and values of μ calculated.

62. Convex Lens.

APPARATUS.—Convex Lens and Holder, Large and Small Pins, Corks, Metre Rule, Small Card.

OBJECT.—The object of this experiment is (1) to find the focal length and power of a convex lens, and (2) to observe the images produced by it under various conditions.

Theory.—If a small object is placed on the axis of a lens, its image will be on the axis also. Let all distances be measured from the lens, and if against the incident light be considered positive, if with it negative. Thus the focal length of a convex lens is negative since it changes a parallel beam into a convergent one. Occulists and opticians, however, use the opposite sign and measure the power of a lens by the reciprocal of its focal length. Thus a lens whose focal length is $-$ 100 cm. is said to have a focal power of $+$ 1 dioptrie, and a lens whose focal length is $-$ 50 cm. is specified as $+$ 2 dioptries. Hence, denoting focal powers in dioptries by D and adding its value to the usual equation for lenses, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = -\frac{D}{100} \quad . \; . \; . \; . \; . \; . \quad (1)$$

where f denotes focal length, u and v the distances of object and image, all in cms. When real images are obtained with a convex lens the value of v is negative. This often causes trouble to students and leads to confusion. Hence, if preferred, the following notation may be adopted.

Let $f = -F$, $u = a$ and $v = -b$. Then equation (1) transform into the working formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{F} = \frac{D}{100} \quad . \; . \; . \; . \; . \; . \quad (2)$$

Here everything is positive and for *real* images with a convex lens all the distances are taken as positive, so only their numerical values are needed. For illustrations see Fig. 21, in which the capital letters denote the points whose distances are given by the corresponding small letters.

Graphical Relation.—It is seen that a, b, and F have the same relation as u, v and f for a concave mirror, hence the same graphical relation will apply.

Focal Lengths and Powers by Simple Methods.—I. Using a distant object as source of light, focus a sharp image of it by the lens on the card beyond. The distance between

lens and card which secures this is clearly the focal length. The negative sign is prefixed if the value of f is sought by (1), whereas it is left positive to represent F in (2). Thus if the numerical value were 12·5 cm., we should find $f = -12{\cdot}5$ or $F = +12{\cdot}5$, and focal power is given by $D = +8$.

II. Place the lens upright in its holder and use a pin on a cork as the object. Look from the other side of the lens with one eye closed, and place a second pin on a cork so that the image of the first seems to stand inverted on it.

Adjust till the image and pin refuse to separate on moving the head sideways. For this remember the golden rule, that the *farther* of two things seems to move *with* the observer, relatively to the nearer thing. Then measure the distances a and b; insert in equation (2) and find F and D.

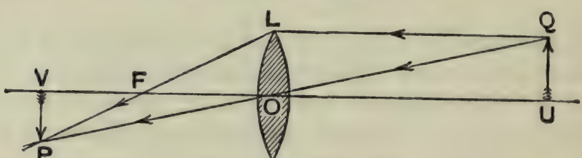

FIG. 21.—Convex lens.

III. SIZE AND POSITION OF IMAGES.—If q be the length of a small object perpendicular to the axis of the lens and p that of its image, we have

$$\frac{q}{p} = \frac{u}{v} = -\frac{a}{b} \qquad \dots\dots (3)$$

see Fig. 21.

This should be confirmed by the use of two pins side by side for the object, the image being inverted on other pins rightly placed as in the experiment with the concave mirror. Several pairs of values of a and b should be taken, equation (3) tested for each such pair, and F and D found from each also.

RESULTS.—Tabulate all your values, and plot a diagram also, showing the value of the focal length obtained by that method.

63. Concave Lens.

Apparatus.—Concave Lenses in Holders, Auxiliary Convex Lens in Holder, Large and Small Pins, Corks, Metre Rule.

Object.—To find the focal length of a concave lens by the use of an auxiliary convex one.

Method.—Referring to Fig. 22, place the auxiliary lens A in a convenient position and a large pin, Q, behind it.

Then, by a second pin, locate and note the position of U, the image of Q.

Next, leaving A and Q unchanged, introduce the concave lens L in front of A, but in the light converging to U. This lens will cause the light to be less convergent and to reach some other point, say V. Locate this new image at V seen by

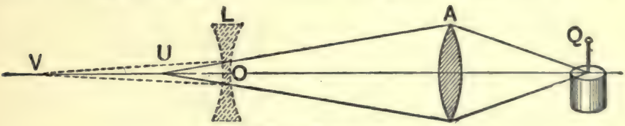

Fig. 22.—Concave lens.

looking at Q through both L and A. Note its position and that of O the centre of the lens L. Then obviously OU and OV are the *numerical* values of u and v the distances of the object and image from the concave lens. For the point U, to which light converges when L is away, is like an actual object for the concave lens L when it is placed as shown. But both these quantities u and v are negative, since they pass from the lens in the direction of the incident light. Hence, the values of the focal length, f cm., and the focal power, D dioptries, can be found from the usual equations for lenses,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = -\frac{D}{100}.$$

It may be noted here that f will come positive and D negative. Thus, if $v = -20$ cm. and $u = -10$ cm., we should find $f = +20$ cm. and $D = -5$ dioptries.

Exercises and Results.—Test the various lenses provided, tabulating all observations and values found for focal lengths and powers.

64. Optical Bank.

Apparatus.—Optical Bank and Accessories, Concave Mirrors, Two Convex Lenses and one Concave Lens.

Objects.—To find the focal lengths of the given mirrors and lenses.

Theory.—Using the same notation and convention of signs as previously adopted, the theory here needed may be written as follows.

Concave mirror ... $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=\frac{2}{}$ . . . . (1)

Convex lens with real images ... ... $\frac{1}{a}+\frac{1}{b}=\frac{1}{\mathrm{F}}=\frac{\mathrm{D}}{100}$ . . . (2)

Combination of two thin lenses close together... $\frac{1}{\mathrm{F}_1}+\frac{1}{\mathrm{F}_2}=\frac{1}{\mathrm{F}}$ . . . . (3)

or $\mathrm{D}_1+\mathrm{D}_2=\mathrm{D}$ . . . . (4)

Symbols.—In equation (1) u and v are distances of object and image from concave mirror and f its focal length, and r its radius of curvature. For the convex lens, F denotes the numerical value of the focal length in cm., D its focal power in dioptries, a and b the distances of object and image both reckoned positively though on opposite sides. In equations (3) and (4) the letters without subscripts denote the focal length or power of the combination, and those with subscripts the same quantities for the component lenses.

Optical Bank.—The optical bank presents a more formal method of dealing with mirrors and lenses, and in the simple form here adopted serves as an introduction to advanced optical work of like character. It consists essentially of a graduated base upon which may be placed various standards. These standards hold in a convenient manner the mirror, lens,

or lenses in use, also the illuminated object and the screen for the image (when real).

Gauge Standard.—If the zero mark on the base of each standard were exactly in line with the object, lens, etc., carried by it, the distance between such object and lens, etc., would clearly be the difference between the readings at the bases of the standards in question. It must not, however, be taken for granted that this condition is fulfilled. If, after examination, there seems any doubt on the point, estimate the distances as follows :—

Disregard entirely the readings at the base of each standard carrying lens, object, etc., but, when they are adjusted, bring up to each in turn the pointer or gauge wire carried on a *separate* standard. As each standard is thus gauged, note the reading at the base of the *gauge standard.* Then the differences of these readings will give the distances required. Thus, if the readings of the gauge standard were 100, 80, and 40 for the screen, lens and object respectively, we should have

$$100 - 80 = 20 = b,$$
$$80 - 40 = 40 = a.$$

Other ways of reading are sometimes found convenient ; also the gauge standard may be used to test the accuracy of the zero marks on the other standards. These methods are, however, left to the oral explanation of a demonstrator.

Exercises.—(*a*) Find the focal lengths of the given concave mirrors by equation (1) both with $u > v$ and $v > u$.

(*b*) Find the focal lengths and powers of each of the given convex lenses, using equation (2) and taking various values of a and b.

(*c*) Find the focal length of a combination of both convex lenses in contact (in the same holder) and note if your values are in accord with equations (3) and (4).

(*d*) Find now the focal length of a convex combination of a convex and a concave lens in contact (in the same holder). Then, using equations (3) and (4), deduce the focal length and power of the concave lens used.

RESULTS.—Tabulate all results neatly, use also the graphical method for focal lengths described in Experiments 57 and 62.

65. Refractive Index by Lens.

APPARATUS.—Convex Lenses and Holders, Pins, Corks, Metre Rule, Card, Calliper Gauge (or Spherometer).

OBJECT.—The object of this experiment is to find the refractive index of the glass of a convex lens by means of its focal power, dimensions, and the known relation between these quantities.

THEORY.—The relation between the focal length and other constants of a lens is usually written

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \quad . \quad . \quad . \quad . \quad (1)$$

where f is the focal length, μ the index of refraction, r and s the radii of curvature of the first and second faces of the lens, each distance being measured from the centre of the lens and reckoned positive if against the incident light and negative if with it. The following form is, however, more suitable for the present experiment.

$$\frac{1}{F} = \frac{D}{100} = (\mu - 1)\frac{8t}{d^2} \quad . \quad . \quad . \quad . \quad (2)$$

Here d is the diameter of the lens and t the *excess* of thickness of its centre over the edge, $F = -f$, $D =$ focal power in *dioptries.*

METHOD.—Find F by any of the methods of the previous experiments. Then either measure d and t by the rule and calliper gauge; or, measure t, or r and s by the spherometer). Insert in equations (1) or (2) and solve for μ the unknown sought.

RESULT.—Tabulate your values of F, D, and μ, stating how F was obtained in each case. Record also the values of d and t or r and s.

66. Refractive Index of Liquids by Mirror.

APPARATUS.—Concave Mirror (or Watch Glass and Black Paper), Card, Clip Stand, Steel Rule, Liquids to test.

OBJECT.—To find the refractive index of a liquid by observations of it in a concave mirror.

THEORY.—Let AB be the surface of a liquid contained in the concave mirror, and let O be such a point on its vertical axis that the image of an object at O is formed at O also, see Fig. 23. Then rays starting from O must, after refraction at A and reflection at D, retrace their original paths. Hence they must be incident normally on the mirror at D. Thus DA produced upward must cut the vertical axis BO in C, the centre of curvature of the mirror. Let MAN be normal to the liquid surface at A. Then the angle of incidence of the ray on the liquid is NAO and equals BOA. Also the angle of refraction of the ray in the liquid is MAD and equals BCA. But, by definition, the ratio of the sines of these angles is the index of refraction. Or, in symbols,

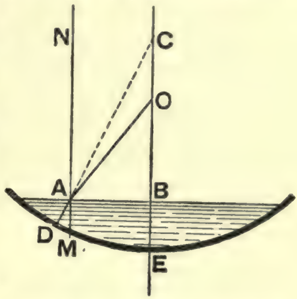

FIG. 23.—Liquid in concave mirror.

$$\mu = \frac{AB}{AO} \div \frac{AB}{AC} = \frac{AC}{AO} \quad . \quad . \quad . \quad . \quad (1)$$

If, now, we use only the central part of the liquid in the mirror, A is very near to B.

We may accordingly write, as our working approximation,

$$\mu = \frac{BC}{BO} \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

METHOD.—With the mirror empty, C is found as the

point where object (a strip of card in the clip stand) and image coincide.

In like manner O is found by coincidence of object and image when the mirror contains the liquid under test.

Thus, measuring the depth of liquid EB at the centre, we find BC, and by (2) the value for μ, the unknown to be determined.

(*Note.*—If, instead of a silvered mirror, only a watch glass is available, place it on the black paper to prevent any reflections from the plane surface of the bench.)

Exercises and Results.—Deal with each of the liquids provided, carefully cleaning the mirror between each filling. Give a sketch showing the theory and practice of the method, and tabulate neatly your observations and calculated values for the refractive indices.

67. Refractive Index of Liquids by Watch Glass.

Apparatus.—Several Watch Glasses, Plane Mirror, Dull Black Surface, Strips of White Card, Clip Stand, Steel Rule, Liquids to test.

Object.—The object of this experiment is to obtain the refractive index of a liquid by putting some in a watch glass, thus making a plano-convex liquid lens.

Theory.—For the present case equation (1) of Experiment 65 may conveniently be written as follows:—

$$\frac{1}{F} = \frac{\mu - 1}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F is the numerical value of the focal length, s that of the radius of curvature of the watch glass, and μ the index of refraction sought. Hence to find μ we need F and s.

Method.—To find s, place the empty watch glass on the dull black surface so that the interior surface of the glass may act like a mirror. Hold over it a piece of card about a quarter of an inch wide, adjusting it till its inverted image

meets it and is the same size. Make sure of the right place by moving the head sideways to see if the image appears to cleave to the object as it should do. It may be found convenient to make the final adjustment by aid of the clip stand. When object and image coincide each is distant s from the mirror. Measure s with the steel rule.

To find F place the watch glass on the plane mirror and pour a little liquid (water or alcohol say) into it, taking care, however, that none is spilt on the mirror. Again hold a strip of card over the watch glass and adjust till the inverted image appears to meet it and to cleave to it. When this position has been obtained the light must have returned on its outward path, but such return from a plane mirror is only possible to a parallel beam incident normally. But, a parallel beam after passing through a convex lens converges to the principal focus distant F from its centre. Thus F is obtained by the steel rule. Note that F is *greater* than s. When the liquid is in, *another* image may be obtained at a distance *less* than s, but this is not the one sought. Hence, having s and F, μ is calculated from (1).

RESULTS.—Use two watch glasses and two liquids in each, tabulate your results neatly, showing all data upon which they are based. Also make a diagram illustrating the theory and practice of the method.

68. Refractive Index of Liquids by Critical Angle.

APPARATUS.—Simple Critical Angle Apparatus or Reflectometer,* White Paper, Rule, Protractor, Liquids to test.

OBJECT.—To determine the refractive index of a liquid by observing the critical angle for the liquid-air surface.

EXPERIMENTAL ARRANGEMENT.—The disposition of the apparatus is shown in plan by Fig. 24. In this it is shown that, mounted on white paper, a cubical glass trough contains the liquid under test, and that, immersed in the liquid, is a

* In London this may be purchased complete for a few shillings.

pair of parallel glass plates, separated only by an edging TT of tinfoil, so as to enclose a very thin film of air between them. A pin, Q, is stuck vertically in the paper behind the

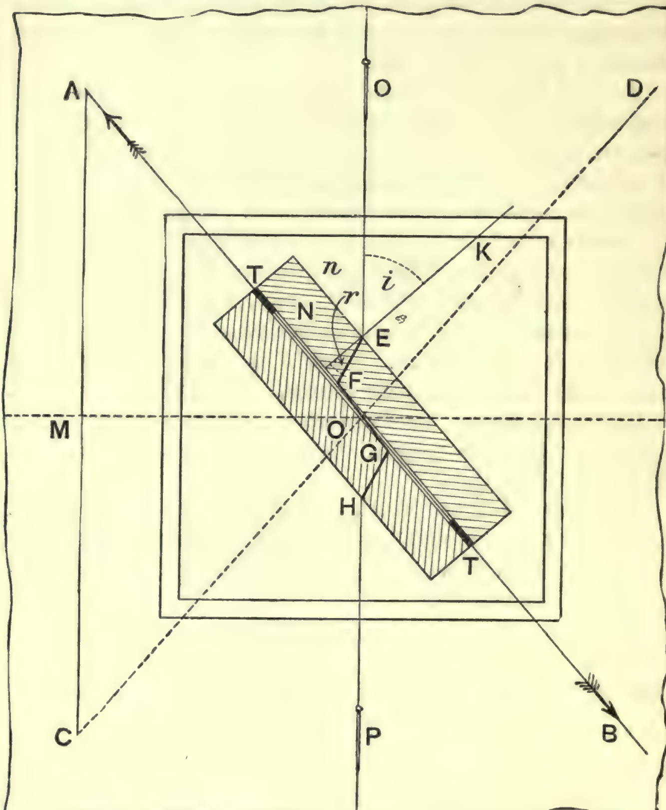

FIG. 24.—Plan of critical angle apparatus in use.

trough, and another pin, P, in front, as shown in the figure. The trough should be set so that its faces are perpendicular to the line QP joining the pins. The apparatus includes a wooden bridge, or stool, which is not shown in the figure.

This stool spans over the trough and controls the pair of plates which fit into an opening in its top. The feet of the stool have pointers, which are indicated in the figure by the arrows at A and B. Hence, by turning the stool the plates are brought to any desired position, and that position is transferred to the paper by the two pointers and may be recorded by marks. The whole should be arranged so that a good light falls upon the paper at the far side AQD of the trough, and *no* specially bright surfaces at each side.

THEORY.—Let the apparatus be in the position shown in Fig. 24, and consider the light passing along QEFGHP. The thickness of the pair of plates and of the air film between them are much exaggerated in the diagram for the sake of clearness. Thus, the path FG in the air film may be taken to represent *grazing* emergence from the first plate and *grazing* incidence on the second. Let n be the refractive index of the liquid and N that of the glass plates. Also let the light in the liquid fall upon the first face of the first plate at the angle of incidence i and be refracted into the plate at the angle r, both with the normal EK. Then, considering this refraction QEF, we have—

$$\frac{\sin i}{\sin r} = \frac{N}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The refraction out of the plate is into air of refractive index practically unity, and angle of refraction practically 90°. Hence, for the refraction EFG, we may write—

$$\frac{\sin r}{\sin 90^\circ} = \frac{1}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus, by multiplication of (1) and (2) we eliminate N and r and find $\sin i = 1/n$, or—

$$n = \frac{1}{\sin i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But if OM be taken at right angles to QE, we see that the angle $MOA = i$. Thus, if a position CD be found for the

plates, of *equal* but *opposite* inclination to the position AB shown, then—

$$i = \text{half the angle AOC} \quad . \quad . \quad . \quad (4)$$

METHOD.—The apparatus being set up as described, turn the stool and the pair of plates till at their far edge a black or dark band appears. Then turn slowly till that band reaches the pin Q when seen in line with the pin P. The pointers then give the position AB, which must be marked on the paper. Next turn the apparatus to the opposite inclination and observe as before, thus determining and marking CD. Then by (3) and (4) n is calculated.

EXERCISES AND RESULTS.—Use the liquids provided, explain carefully, with diagrams and sketches, all you have done, and tabulate all observations and results.

69. Microscope.

APPARATUS.—Several Convex Lenses assorted, Two Millimetre Rules, Two Clip Stands.

OBJECT.—To set up a compound microscope and find its magnifying power.

METHOD.—Choose for the objective the lens of greatest focal power and a weaker one for the eye lens. Hold the objective near one of the rules lying on the table, bring it slowly away until it gives a *real inverted* image of the rule, then place over it the eye lens, adjusting both until a sharp image is obtained. The lenses may be fixed in their right positions by the clip stands. Thus the microscope is set up.

To find its magnifying power view one of the scales through it with, say, the right eye, at the same time viewing (not through the microscope) the other scale with the left eye. This second scale must be at a convenient distance, say 25 cm., from the eye, and so placed that the two scales seen in this way appear to be superposed in the field of vision. Then the magnifying power is the number of unmagnified divisions of

one scale corresponding to one of the same divisions of the other scale as seen magnified through the microscope.

RESULTS.—Set up several different microscopes and obtain the magnifying power of each.

70. Telescope.

APPARATUS.—Several Convex Lenses assorted, Distant Scale, Two Clip Stands, Metre Rule.

OBJECT.—To set up an astronomical telescope and find its magnifying power.

THEORY.—The magnifying power of a telescope is the ratio of the angle subtended at the eye by the image to that subtended by the object seen directly. Thus, if these angles are called $m\theta$ and θ respectively, then m is the magnifying power. Let the numerical values of focal lengths of object-glass and eye-piece be F_1 and F_2 respectively, and let l be the length of the image formed by the object-glass. Then, when the telescope is used for distant objects, this image is practically at the distances F_1 and F_2 from the object-glass and the eye-piece respectively. Thus, neglecting the distinctions between tangents and radians, we have the approximate relations

$$\theta = \frac{l}{F_1}$$

and

$$m\,\theta = \frac{l}{F_2}.$$

Hence

$$m = \frac{F_1}{F_2} = \frac{D_2}{D_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the D's denote the focal powers in dioptrics. It therefore follows that a telescope gives no magnification unless the power of the eyepiece exceeds that of the object-glass.

METHOD.—To set up the telescope, choose suitable lenses, put the eye-piece in a clip stand close to the eye and adjust the object-class also in a stand until the distant scale is sharply focussed. To find the magnifying power of the instrument

arrange that the image of the scale seen by one eye through the telescope appears superposed on the scale seen directly by the other eye. Then note how many divisions of the scale itself correspond to one of its magnified image. This number is the magnifying power sought.

EXERCISES AND RESULTS.—The value thus experimentally found should be compared with that derived from equation (1), the powers of the lenses being found by method I. of Experiment 62. Repeat the entire experiment with another pair of lenses and tabulate all results neatly.

71. Opera Glass.

APPARATUS.—Convex and Concave Lenses assorted, Distant Scale, Two Clip Stands, Metre Rule.

OBJECT.—To set up an opera glass and find its magnifying power.

METHOD.—Choose for the object-glass a convex lens of low power, and for the eye-piece a double-concave lens of as high power as possible. Set the eye-piece in a clip near the eye and adjust the object-glass until the distant scale is sharply focussed. Then, looking at the scale directly with one eye, the magnifying power is the number of its divisions corresponding to one of its image seen through the telescope by the other eye.

EXERCISES AND RESULTS.—Set up several pairs of lenses and find the magnifying power of each opera glass so formed. Sketch carefully a ray diagram and decide whether equation (1) of last experiment should apply to the present case. Also test each combination to ascertain if it is experimentally confirmed.

72. Optical Projection.

APPARATUS.—Condensing and Focussing Lenses, Lamp, Gauze and Slides, Screen, Cardboard or Diaphragms.

OBJECT.—To set up a projection lantern and focus on the screen a magnified image of a slide or other object.

Method.—Set the lenses in clamps at the height of the centre of the lamp. Place the condensing lens, which is of short focus, near the lamp, but so that the rays leaving it form a parallel beam. Place the gauze just in front of the condensing lens. Next place the focussing lens in the path of the rays and adjust it until a clear image of the gauze is seen on the screen. Now substitute the slides for the gauze, noting which way they need to be placed to give an erect image on the screen. Try to improve the image by using diaphragms on the lenses and by screening the lamp. Note also the opposite curvatures produced in the image of the gauze by placing the diaphragm successively before and after the focussing lens.

Exercises and Results.—Describe carefully with explanatory sketches all you have done and observed.

73. Optical Dispersion.

Apparatus.—Lamp, Two Screens and Holders, Two Convex Lenses and Holders, Prism and Stand, Metre Rule and Protractor, Darkened Room.

Object.—To project upon a screen a pure spectrum of the given source and find the refractive indices of the glass for the extreme rays.

Method.—Place the apparatus in the following order—lamp, small screen with vertical slit in, lens, large screen at a distance. Set the lamp to illuminate the slit as well as possible. Adjust the lens to give a parallel beam of light. Place the second lens to catch this beam, and focus the slit sharply upon the screen. Then introduce the prism on its stand between the lenses so as to receive the light on one face and refract it. The prism then deviates the beam and disperses the light, thus giving a spectrum. Without changing its distance from the prism, shift the screen round so as to receive this deviated light normally. Rotate the prism about a vertical axis until further rotation either forwards or backwards *increases* the deviation. You have then obtained the

important position of *minimum deviation*. Introduce the second lens between the prism and the screen, carefully preserving its original distances from each.

If there is any fear that this has not been done, check the position as follows. Remove the second lens and rotate the prism slightly so that one face acts as a mirror, and sends the light to the position of the mean yellow rays in the spectrum of minimum deviation. Then replace the second lens and adjust so as to give a sharp image of the slit on the screen. Now, leaving the lens untouched, replace the prism in its refracting position and again rotate to secure minimum deviation.

A *pure* spectrum has thus been obtained. A spectrum may be obtained by use of a single lens, but it is *not* pure, because the light then falls upon the prism at varying angles, since the beam is not parallel.

By the principle of Experiment 61 the values of the refractive index for any rays, say the extreme ones, may also be found.

Exercises and Results.—Describe carefully with diagrams and tables all you have done and observed, and state also the values of μ found for the extreme rays.

PART IV.—SOUND

74. Interval by Smoke Traces.

Apparatus.—Two or more Tuning Forks, each with an Aluminium Style, Bow and Rosin, Glass and Camphor for smoking it, Steel Rule.

Object.—To find the interval (or *ratio of frequencies*) of any two forks.

Method.—The glass is lightly smoked on one face by holding it in the fumes from burning camphor, and then laid down on the bench with its smoked side up. The two forks may now be grasped by their stems in the left hand, with their *styles down.* With the right hand the two forks are now bowed successively or simultaneously, and are then drawn along, stems foremost, with their styles lightly touching the smoked surface of the glass. Each fork accordingly leaves a sine-graph smoke trace. These traces are each of constant wave-length *if*, in drawing the forks over the glass, the motion of the left band was uniform. But if the motion were not uniform this introduces no error, since it affects the two traces equally. Hence, in any case, the ratio of the wave-lengths, occurring *side by side*, is the *direct* ratio of the *periods* of the vibrations, and is the *inverse* ratio of their *frequencies.*

Exercises and Results.—Take several traces of one pair of forks and, if other forks are available, pair them off in all possible ways, and thus obtain a check upon your values of the various ratios of frequencies, or intervals. Give sketches of your traces and tabulate all your results. If the absolute value of the frequency of one fork is known, state those of the others.

75. Frequency by Fall-Plate.

Apparatus.—Fork and Fall-Plate Apparatus, Camphor for smoking, Bow and Rosin, Thread, Steel Rule.

Object.—To ascertain the frequency of a given tuning-fork by the trace it leaves on a falling-plate. The value of gravity is taken as known.

Method.—The apparatus consists essentially of a smoked glass plate, arranged to fall past the tuning-fork whose pitch is to be determined. The fork is mounted in a block A

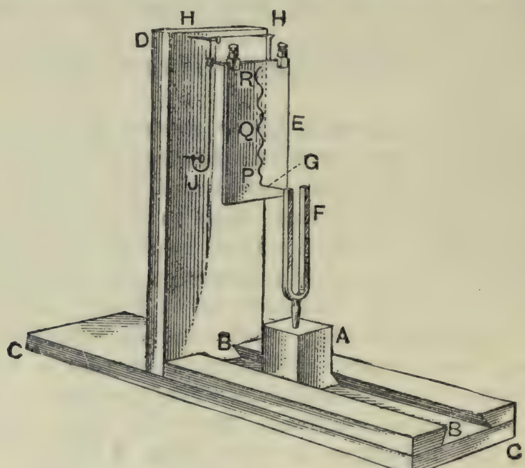

Fig. 25.—Fork and fall-plate.

(Fig. 25), sliding in a groove BB in the base-board CC, which also carries a vertical board D to support the smoked plate E, ready for its fall. The fork F carries in one prong a light style G of thin aluminium foil attached by soft wax (or by a screw). The plate is smoked by the fumes of burning camphor. It is then hung by a fine thread passing over two nails H, H, and under screws at the *back* of the plate (which may be the side pinch screws of terminals used for the plates of Daniell

cells). This method of suspension causes the plate to tilt forward very slightly, like pictures on a wall. Then, on adjusting matters so that, to start with, the style just touches the plate, on releasing the plate by burning the thread, the style will be in gentle contact during the fall. To prevent breakage of the plate, a second loop J, of stronger thread, may also be used, passing round a lower pair of nails on the board, and attached to the terminals on the plate, so as to arrest it at the end of its fall.

These details being arranged, the fork is set vibrating by the bow, and the thread burnt between the nails H, H. Then, as the plate falls, a wavy trace is marked upon it, which is clearly the resultant of a simple harmonic motion horizontally, and a uniformly accelerated motion vertically. The acceleration g being assumed, we have thus the data for calculating the frequency N of the fork.

THEORY.—But, owing to the very slow motion of the plate near the commencement of its fall, the corresponding wavy trace on the smoked surface will be very crowded, thus rendering a counting of the waves practically impossible. It is accordingly advisable to avoid this region and proceed as follows: Select two portions, PQ and QR, where the waves can be readily counted, and each containing the *same* number n of waves, the lengths PQ and QR being l_1 and l_2 respectively. Further, let $t = n \div \mathrm{N}$, which is the time of fall from P to Q or Q to R. Then, denoting by u the speed of the plate when P passes the style, we have by elementary kinematics—

$$l_1 = ut + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But the speed as the point Q passes the style, is $u + gt$; hence for the portion QR we have

$$l_2 = (u + gt)t + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad (2)$$

Thus, by subtraction, we find

$$l_2 - l_1 = gt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Whence, on writing for t its value, we have

$$N = n\sqrt{\frac{g}{l_2 - l_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Exercises and Results.—Test the forks provided, making several traces for each. Write a careful account of your work, and tabulate your observations and results.

76. Sonometer.

Apparatus.—Sonometer with Movable Bridge, Pulley and Wrest, Piano Wires of various diameters, Kilogram Weights and Blocks to hold sonometer down, Micrometer Gauge, Tuning Forks of frequencies 128, 256, and 384 per second.

Object.—To ascertain how the frequency of a "string" depends upon its linear density, tension, and length.

Method.—Let the frequency be N, the linear density m, the tension or stretching force F, and the length l (between the bridges). Then, in seeking to ascertain how N depends upon m, F, and l, we must not vary these quantities all together. For, in that case, their several contributions to the change in N would be blended and their separate values remain unknown.

Frequency and Length.—Let us therefore begin by changing only one of these variables, namely l, and so find what changes in it correspond to certain changes in N. We are accordingly to keep m and F constant. That is, we keep to a given "string," stretched with a given force F. Hence, choose a wire, mount it on the sonometer, and then tune the entire length between the fixed bridges to your lowest fork.

The final tuning is best made by striking the fork on the palm of the hand (or other soft substance), plucking the string and listening for the waxings and wanings of the intensity, called "beats." Adjust the tension by the wrest till the beats fade away. The slower the beats the better the tuning, which is satisfactory only when the beats cease to be audible. Next,

leaving the same string at the same tension, insert the movable bridge to isolate a given length, and vary this length till it gives vibrations in tune with the second fork. Note this length carefully. Similarly determine and note the length in tune with the third fork.

Record these observations in tabular form, stating in one column the quantities (F and m) kept constant, and in others the variables (N and l) and their corresponding values. Then find *what combination* of these variables retains a *constant* value for all the cases observed. To find this combination, try the product, quotient, product or quotient of one quantity by the square root of the other, etc., till the right result is reached, then note it in another column. This shows how N depends upon l, and so completes the first part of the task.

TENSION AND LENGTH.—Let us next find how N depends upon F. Shall this be done by changing F to tune the string to each fork in turn, keeping m and l constant? This would be possible, and might seem very simple if a complete series of small weights were available. But it would not be so convenient as adjusting l to effect the tuning. Hence, it is now best to keep m and N constant, make F some exact number of kilograms by using a wire passing over the pulley, and then tune as long a string as possible to the lowest fork, noting the force F and length l. Next alter the F to some smaller value, and change the l till the string is again in tune with the *same* fork, noting the F and l as before. In this way several sets of values may be obtained, and again a *combination* of the variables found which remains constant while they vary. Tabulate all as in the first part.

LINEAR DENSITY AND LENGTH.—Lastly, we may keep N and F constant, but change m, and find how l must be changed to preserve N unchanged with a thicker or thinner wire. It is obvious that with wires of the same material, m varies as the square of the diameter. To obtain numerical values of m, the density of the steel wires may be taken as 8 gm. per c.c. Tabulate all as previously and again find a combination of the variables which remains constant.

FINAL RELATION.—We have now obtained three relations between the length l and the other quantities N, F, and m, showing how pairs of these quantities may yield combinations which remain constant under certain circumstances. Hence these may be combined to give a final expression which will remain constant under any circumstances. This expression should now be deduced, when, if all is correctly done, the results will be in accord with the following summary.

Constants.	Variables.		Combination which remains constant.
F and m	N	l	Nl
m and N	F	l	$\frac{l}{\sqrt{F}}$
N and F	m	l	$l\sqrt{m}$
None	All		$\frac{Nl\sqrt{m}}{\sqrt{F}}$

Or otherwise, the final results show that

$$N \propto \frac{\sqrt{F}}{l\sqrt{m}}$$

And this truly expresses the dependence of frequency upon the other quantities as desired.

RESULTS.—Describe carefully with sketches and tables all you have done and deduced.

77. Frequency by Beats and Sonometer.

APPARATUS.—Sonometer, Two Tuning Forks of the same pitch or nearly so, Wax and Lead Shot.

OBJECT.—To obtain the frequencies of the given forks.

Method.—Stick shot on the prongs of one fork with wax till it gives beats with the other, say about four per second. Carefully count these beats for about ten seconds and so find their number b per second. Then we may write

$$N \sim N' = b \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where N is the frequency of the unaltered fork, and N′ that of the loaded fork, since it is known that the number of beats per second is always the difference of the frequencies.

Next compare the forks with a given string on the sonometer, and find the lengths l and l' of string which are in tune with each. Then, since we have found in a previous experiment that the product, frequency into length, is a constant for a given string, we have

$$Nl = N'l' \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We accordingly obtain from (1) and (2)

$$N = \frac{bl'}{l' \sim l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the two forks were originally in unison, this value found for the unaltered frequency completes the determination.

If they were originally different, the loaded one must now be stripped of shot and wax, the other loaded and the operation repeated, when the frequency of the other, M say, may be in like manner determined.

It is evident from the right side of Equation (3) that great care must be exercised in finding each l, since only their *difference* occurs in the expression.

Results.—Tabulate your observations and state your values for the frequencies.

78. Frequency by Vertical Monochord.

Apparatus.—Vertical Monochord (with Movable Bridge, and Kilogram Weights), Piano Wires, Tuning Forks, say about 128, etc., Balance and Weights, Metre Rule.

OBJECT.—To find the frequencies of the given forks, assuming the expression for that of a "string."

THEORY.—It is known that the frequency of a vibrating string is given by

$$N = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

where l is the length of the vibrating part between the bridges, F the stretching force, and m the mass per unit length. If the force F is due to the weight of a mass M gm. we have $F = Mg$, where g is the acceleration due to gravity, say, 981 cm. per sec.^{2}

METHOD.—Set a wire in position in the monochord, use the movable bridge at about the full distance, and add kilogram weights to stretch the wire till it is nearly as sharp as the fork, then finally adjust to tune by moving the bridge. Note the weights used and the length. Then remove the weights, take off the wire and weigh a measured length of it, and so calculate m, the mass in gms. per cm. of length. Thus, inserting the values of l, F, and m in the equation, the frequency N is readily calculated.

Repeat the process for each fork provided.

RESULTS.—Tabulate your observations and results, explaining all you have done.

If these forks have been dealt with previously by another method, compare the results so obtained with these.

79. Frequency by Vibrating Strip.

APPARATUS.—Thin flexible Steel Strip, say 40 cm. by 2 cm., Vice or Clamp to hold it, Tuning Forks (say about 128, etc.), Metre Rule.

OBJECT.—To find roughly the frequencies of the given forks, assuming that those of a strip vibrating transversely are inversely as the *squares* of its vibrating lengths.

METHOD.—Notice the number of ticks per second made by

your watch ; suppose it is *four* exactly. Clamp the strip with a long free end and set it in vibration. Adjust the length of the vibrating part till it just keeps time with the watch-ticks. Measure this length l and note it. Next set the strip so that a shorter length L is free to vibrate. Adjust this length L till it gives vibrations in tune with one of the forks of frequency, N, say.

Then, by the assumed law for the strip, we have

$$NL^2 = \text{a constant} = 4l^2.$$

Whence N may be calculated since the L and l are both known.

Exercises and Results.—In this way estimate the frequency of each fork, and check, if possible, by listening to the forks in succession. Note, also, if the lengths for forks of known frequencies agree with the law.

80. Frequency by Siren.

Apparatus.—Siren (of Cagniard de la Tour's form), Tuning Forks and Organ Pipes of frequencies 256 or more per second, Rubber Tube and Screw Clip, Foot Bellows and Wind Chest with weighted Valve.

Object.—To find the frequencies of the given forks and pipes.

Method.—This siren consists essentially of a vertical spindle carrying a disc pierced with a ring of holes which alternately open and close a similar ring of holes on the top of the cylindrical chamber. At the upper part of the apparatus is added a counting mechanism with fingers and dials which record the number of rotations of the disc. The two rings of holes have opposite obliquities so that the pressure of the air as it escapes through them spins the disc. Thus, by suitable adjustment of pressure, the siren may be tuned to any desired note within certain limits.

To determine the frequency N of a given sound by the siren, we must accordingly (1) tune the siren to the sound in

question and keep it in tune; (2) observe the indications of the dials at the start and finish of a tuned period of t seconds and thus find the number r of complete rotations effected by the disc. Also count the number n of holes in the ring. Then, evidently, we have

$$N = \frac{nr}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

PRECAUTIONS.—But some care is necessary both in the tuning and counting. For example, at some pressures the siren is distinctly flattened by putting the counting mechanism into gear. Hence, though the siren is usually provided with a sliding motion to put the counters in and out of gear, it is safer to keep them in gear throughout the experiments. If available a stop-watch may be used in timing, when the counting mechanism is always kept in gear.

For the tuning, the final adjustment of pressure should be made by the screw clip on the rubber connecting tube. The screw may easily be moved by only an eighth of a turn at a time, and thus the air pressure and the pitch of the siren slightly changed. But it should be noted that the siren will respond so slowly to such a slight change of the pressure that the pitch may be falling or rising almost *imperceptibly* for *many* seconds. Hence, after touching the screw, allow a timed interval of at least a *minute* to elapse before assuming the siren to be steady enough to be worth comparing your fork with it.

In a preliminary adjustment of the wind pressure by the screw, when the pitch is rapidly altering, the experimenter may often feel uncertain whether the siren is sharp or flat, or by how much. It is then convenient to touch the disc or the spindle very lightly with the finger, thus decreasing its speed and so lowering the pitch. By this means the siren, if too sharp, can be brought down to the pitch desired; or, if already too flat, the fact immediately ascertained.

If by touching the spindle with a feather the siren can

be slowed so as to give a total number b of beats with the fork in the time t, the siren preserving all the time the *sharper* note, we should have for the fork,

$$N = \frac{nr - b}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The beats should be kept at about four per second and care taken that they never vanish.

Exercises and Results.—Test the forks and pipes provided, tabulating your results and explaining carefully the routine followed out.

81. Speed of Sound.

Apparatus.—Adjustable Water Resonator (or Stick, Rule, and Tall Jar of Water), Forks of known frequencies (say 256, 384, and 512), Thermometer.

Object.—To find the speed of sound in air at the room temperature.

Method.—Adjust the level of water in the apparatus until the air column above it is very short, and increase it till it responds as strongly as possible to one of the forks set in vibration and held close over. In the adjustable resonator this adjustment is made by sliding the water reservoir (see p. 318 of Barton's *Sound*) up and down and thus raising and lowering the level of water in the tube. If only the jar and stick are available, the jar is part filled with water and the stick pushed in to raise the level of the water or withdrawn to lower it again.

With either form of resonator the level should be carefully adjusted several times, the lengths measured and the mean taken.

Theory and End Correction.—Let v be the speed of sound in air at the room temperature $t°$ C., and let l be the length of the column of air which resonates best with a fork of frequency N.

Then $$v = N\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ is the wave length in air of the fork's musical sound. If the column of air is very long compared with the radius r of the bore of the tube, we may take λ to be $4l$ nearly, for a rough value. But it is better to make an end correction to the air column to obtain the true quarter wave length. This correction is practically $\cdot 06r$, and needs adding to l. We may thus write

$$\lambda = 4(l + 0\cdot 6r) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, substituting in (1), we have

$$v = 4N(l + 0\cdot 6r) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Again, it is known that

$$v = v_0 \sqrt{1 + at} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where v_0 is the speed of sound at 0° C. and a is the coefficient of gas expansion.

Thus the experimental result calculated from (3) may be compared with that from (4), taking $v_0 = 33{,}200$ cm./sec. nearly, and $a = 1/273$ or $0\cdot 003665$ per 1° C.

If one of the forks allows of two positions of resonance being found at lengths l_1 and l_2, then we have

$$\lambda = 2(l_2 - l_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For these two lengths are respectfully *one* quarter of a wave length minus the end correction and three quarters of a wave length minus the end correction.

Exercises and Results.—Use each fork to find the speed of sound in air, tabulating all observations and comparing your experimental values with that theoretically determined from the observed temperature and the known constants.

Eliminate the end correction by two positions if possible, as in (5). If not possible, add the end correction as shown in (3).

PART V.—MAGNETISM

82. Magnetic Fields.

APPARATUS. — Three Bar Magnets, One Horseshoe Magnet and Keeper, a small Pivoted Magnet, Iron Filings in Sprinkler, Four Sheets of Quarto Paper and one of Stiff Cardboard.

OBJECT.—To find the qualitative characters of those special regions, or *magnetic fields*, round various magnets or combinations of magnets.

LINES OF FORCE.—This is here done by finding in each case a number of *lines of force*. These are lines whose directions at any point are those along which magnetic poles placed there would be urged by the field. The direction in which a north pole would be urged is called the positive direction along the line, and should be indicated by an arrow. It is called the direction of the magnetic field. The south pole is urged in the opposite or negative direction.

MAGNETIC NEEDLE AND FILINGS.—Thus, if a pivoted magnet, or magnetic needle, is placed in a magnetic field, it tends to set itself along the lines of force of the field there, the north pole indicating the positive direction of those lines. If very freely pivoted, the needle will vibrate some time before settling in the right direction. If not very free to turn, it may soon settle but not quite in the right direction.

It is therefore always better to tap the table near the needle to ensure its settling properly.

This tapping, desirable when the needle is used, is indispensable when iron filings are employed as indicators of the

lines of force. For the filings are not pivoted at all and only become feeble magnets because placed in the magnetic field.

METHOD.—By placing the card and paper over any magnet or combination of magnets, the corresponding field may be ascertained and shown by sprinkling a few iron filings on the paper and tapping lightly with the finger or the end of a lead pencil on the card at a place where it rests on the magnet. The pivoted magnet may be used to show the positive direction of any line. The character of each pole of the magnets used should be noted also.

EXERCISES.—Obtain the lines of the fields due to the following arrangements and carefully copy each one in your note book. Do the filings show in any way where the field is strongest? Give reasons for your answer.

(*a*) One straight bar magnet.

(*b*) One horseshoe magnet, with keeper off.

(*c*) Two bar magnets side by side about 8 cm. apart, unlike poles near each other.

(*d*) The same but with like poles near each other.

(*e*) One magnet with the soft iron keeper near.

(*f*) Two bar magnets arranged nearly like the letter **T**, but with 2 cm. space between them.

(*g*) Three bar magnets arranged nearly like a triangle, but with the ends about 2 cm. apart.

83. Magnetic Declination and Dip.

APPARATUS.—Vibration Apparatus (consisting of rectangular box with glass top, torsion head, silk suspension and brass stirrup), Bar Magnet and Brass Bar, each to fit the stirrup; Cardboard, Drawing Pins and Protractor; Work Bench with geographic meridian marked on it.

Dip Circle and Needle, two Bar Magnets (Spirit Level desirable, if not on Dip Circle).

DEFINITIONS.—The declination is the angle between the geographic and magnetic meridians. The *dip* is the angle between the horizontal plane and the lines of force of the

earth's magnetic field. In estimating these angles the northerly direction of each plane or line is taken.

Declination.—Hence to determine the declination we have to find the magnetic north, which we do as follows, using the vibration apparatus. First we must make sure there is no twist on the silk fibre. This is done by placing the brass bar in the stirrup and adjusting the torsion head till the bar settles along the centre line of the box or swings equally on each side of that line. Secondly, take out the brass bar and put the bar magnet in the stirrup. Pin the cardboard on the work bench and mark on it geographic north. Remove all magnetic materials to a distance. Place the vibration apparatus upon the card, adjusting the direction of the box until the magnet sets exactly along the centre line (marked on the base of the box) or swings equally on each side of that line. When in this position mark in pencil on the card the direction occupied by the side of the box and indicate by an arrow the direction which seems to be magnetic north. Now, if the magnetic axis of the bar magnet coincided with its geometric axis, the line just marked on the card would be the real magnetic north. But, as this coincidence seldom occurs and cannot be assumed, we must now take the magnet out of the stirrup and replace it upside down, but keeping the ends of the bar at the same ends of the box as before. Repeat the adjustment of the box till the magnet is central and again mark a line on the card along the side of the box showing apparent magnetic north. These two lines will probably intersect, or may be produced to do so. Bisect the angle between these two apparent norths and indicate along this bisector by an arrow which is real magnetic north. If the student cannot understand how this is the true magnetic north, he must ask a demonstrator.

Finally measure with the protractor the angle between the true north and the magnetic north, noting whether the latter is west or east of the other. This angle is the declination, and should be recorded as west or east declination according to what has been observed.

Dip.—Set the dip circle with its face in the magnetic meridian already found. Level it, place the dip needle in position and allow it to settle. Read *both* ends of the needle (if the apparatus allows of this). Remove the needle, and replace it with its other face towards the dip circle, leaving the latter unmoved. When the needle has settled, again read both ends. Take the needle out and carefully reverse its polarity by about twenty or more strokes of the two magnets provided. Use the needle as before, taking four readings with it in its new state. These make eight readings in all, the mean of which must be taken for the final value of the dip.

Why are all these readings needed? If you do not know, and cannot find out by thinking, ask a demonstrator.

Results.—Write an account of your procedure and the reasons for it, giving the observations made and the values finally deduced for the declination and dip at the given place and time.

84. Magnetic Balance.

Apparatus.—Hibbert's Magnetic Balance with other magnets and weights.

Objects.—To confirm the *inverse square* law, to *compare* the pole strengths of long magnets, and to determine those pole strengths *absolutely*.

Description of Balance.—The balance devised by Mr. Hibbert for the above purposes is shown in Fig. 26. In this AB is a magnet pivoted at the holder C; a known weight W can slide along CA, and its distance a from C may be read off on a scale behind the magnet. A magnet holder E slides up and down a graduated upright BD, on which the distance d from B to E may be read. In reading either of these scales the student must carefully avoid looking obliquely, as this involves error.

Exercise 1. Inverse Square Law.—In the holder E place a magnet so as to repel B and adjust its position until

its end overlaps that of AB by 2 or 3 cms. The reason for this is that the poles of the magnets in use are a cm. or more from the ends of the magnets. Next slide the weight W until it balances this repulsive force as shown by AB settling horizontally. The adjustment of position of magnet in holder E

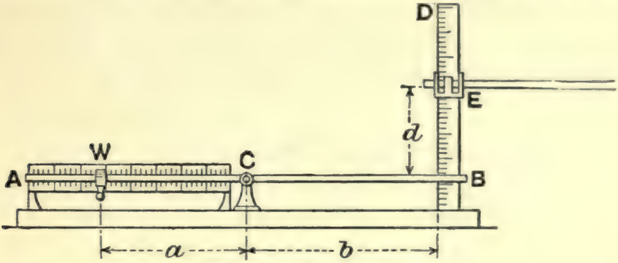

FIG. 26.—Hibbert's magnetic balance.

may now be varied till W needs to be as far as possible from C. Then, if F is the force between the two poles vertically over each other and near B and E, we have, by the theory of the lever

$$a/b = F/W \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus, since b and W are constants, a is proportional to F, and may be taken as a measure of it.

Take various values of d and obtain the corresponding values of a, recording all in the columns of a table. Add other columns headed d^2, $1/d^2$, and ad^2, calculate and fill in the required values.

Plot a graph with the values of a as ordinates, and those of $1/d^2$ as abscissæ. Consider carefully its meaning, also that of the column headed ad^2.

Do the graph and the table confirm Coulomb's law that like magnetic poles repel with a force inversely as the square of their distances apart?

EXERCISE 2. POLE STRENGTHS COMPARED.—Place the magnet X in the holder E at height d, and adjust the magnet to obtain the strongest repulsion. Slide the weight W to the

balance point, and read the distance a_1, say. Then if m is the pole strength of AB, and m_1 that of X, we have

$$\frac{a_1}{b} = \frac{mm_1/d^2}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Next, keeping the magnet AB in place and d unchanged, substitute for X the magnet Y with pole strength m_2 say, and needing W at a distance a_2 to balance the repulsion.

Then

$$\frac{a_2}{b} = \frac{mm_2/d^2}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence by (2) and (3) we find

$$\frac{a_1}{a_2} = \frac{m_1}{m_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus the ratio of the a's measures that of the pole strengths of X and Y which was to be found.

EXERCISE 3. POLE STRENGTHS DETERMINED.—To find the absolute values of m_1 and m_2, the pole strengths of X and Y, proceed as follows. Leaving Y in the holder E, substitute for AB the magnet X, fixing it in the holder C ; the two magnets being so arranged that repulsion occurs at BE. Adjust the weight W to distance a to obtain the balance, and note the height BE = d, say. Then, in the usual units,

$$\frac{a}{b} = \frac{m_1 m_2/d^2}{W}$$

or

$$m_1 m_2 = \frac{Wad^2}{b} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In these equations the distance d should be in cms., and the weight W must be expressed in dynes.

Then, multiplying equations (4) and (5) together and taking the square root, we find

$$m_1 = \sqrt{\frac{Wad^2a_1}{ba_2}} \quad . \quad . \quad . \quad . \quad (6)$$

We may then obtain also

$$m_2 = \sqrt{\frac{Wad^2a_2}{ba_1}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

DISTURBING INFLUENCES.—It should be noted that all the results with this apparatus are slightly affected by the small oblique forces exerted by the distant poles, of which we have here taken no account. This disturbance is almost entirely removed in the experiment on the field of a single pole (Experiment 85).

RESULTS.—Present clearly your observations and the conclusions drawn from them in each of the three parts of the experiment.

85. Inverse Square Law by Single Pole.

APPARATUS.—Magnetometer with upright and holde. sliding on it, Long Bar magnet with balls at its ends.

OBJECT.—To confirm the inverse square law for the field due to a single magnetic pole.

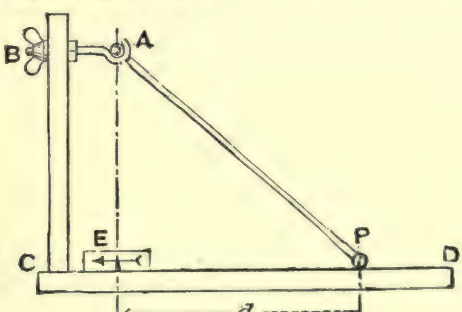

FIG. 27.—Field of single pole.

METHOD.—The bar magnet is removed far from the magnetometer, which is then set so that the needle points at right angles to its bar, CD, in Fig. 27. The bar magnet AP is then placed with one pole, A, resting in the holder

B, so as to be vertically over the centre of the needle, the other pole, P, being in place on the bar CD, all as shown in the figure.

Then the distance d between the centre of the compass needle E and the pole P is read on the scale and recorded.

The deflection θ of the needle E is also read on the graduated circle. Then this deflection is due almost entirely to the pole P, since A causes practically no deflection of the needle, being vertically over its centre. Now examine in what way this deflection θ depends on the distance of the pole P.

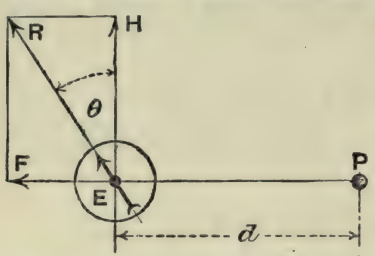

Fig. 28.—Deflection due to pole's field.

Theory. — Referring to Fig. 28, which shows a plan of the compass E and pole P, let the field of P at E be denoted by F, and shown to scale by EF. Call the earth's field H, and show it to scale by EH. Then the two compound, by the parallelogram law, to the resultant field represented by ER, along whose lines the needle settles at an angle θ to its original position. Thus, we have

$$F = H \tan \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

showing that the values of $\tan \theta$ measure the F's for different distances d.

Exercise and Results.—Vary the distances d, taking five or six positions, but avoiding those in which the pole is very near the compass. Note them and the corresponding angles of deflection θ. Tabulate the values of d, θ, $\tan \theta$, and combinations of d, θ, etc., to obtain, if possible, some quantity that remains *constant* while d varies. Also plot curves in the ways you think best to find the law of the field with distance. What is your conclusion on the matter? Since $F = m/d^2$, where m is the strength of the pole P, find m from your

results. If you do not know the value of H for the place where the experiment is done, ask a demonstrator.

86. Field of Bar Magnet.

APPARATUS.—Magnetometer and two Bar Magnets.

OBJECT.—(1) To find the ratio of the axial field F of a bar magnet to that of G at the same distance d from the centre, but sideways, how each diminishes with distance d, and (2) to compare for two magnets their *magnetic moments* M, where M for a magnet is the product of strength of either pole into the distance between the poles.

1. AXIAL AND EQUATORIAL FIELDS: THEORY.—Consider any point P along the axis OP of a bar magnet and any other point Q on a line drawn through the centre O of the magnet, and perpendicular to its axis. See Fig. 29. Then it follows

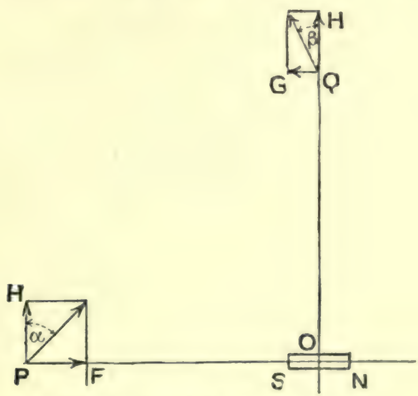

FIG. 29.—Field of bar magnets.

from the symmetry of the case that the fields at P and Q are each *parallel* to the axis. It may also be seen, by tracing a line round, that these fields are in *opposite* directions.

Let us now suppose that OQ is magnetic north, OP magnetic west, and that at P a compass needle is placed, and is

deflected through an angle α by the axial field of strength F due to the magnet. Then by the figure we have

$$F = H \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where H is the strength of the horizontal component of the earth's field. The fields F and H are shown to scale by PF and PH in the figure.

Again, if the compass were placed at Q, the deflection β would occur owing to the field G, where

$$G = H \tan \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus the tangents of the angles of deflection α and β in the two cases measure the fields F and G of the bar magnet. These may be called the axial and equatorial fields respectively.

The relations of the compass at P and Q to the bar magnet constitute the first and second principal positions of Gauss. They are often called the A and B tangent positions.

METHOD.—By using the above two positions, and giving to the distances OP and OQ a series of equal values, find the relation between the axial and equatorial fields at the same distance, and also how each field diminishes with the distance. Does the inverse square law hold good in these cases; if not, what is the new law?

Tabulate d, $\tan \alpha$, $d^3 \tan \alpha$, and the same for β.

To find the deflection in each case, the bar magnet should be placed at the required distance, both ends of the needle read, first tapping it if necessary to prevent sticking. The bar magnet should then be reversed, north and south poles changing places, both ends of the needle again read; then four more readings obtained with the magnet at the other side of the compass. The mean of the eight readings gives the required value.

Note.—In all the positions of the magnet hitherto dealt with, its axis lies along the *magnetic east and west line.* It is only OQ that lies magnetic north and south.

2. Magnetic Moments.—To compare the magnetic moments of two magnets, use them in turn at the *same distance* from the compass and in the *same manner*, and take the *ratio* of the *tangents* of deflection to be a measure of the *ratio* of the *magnetic moments* of the magnets. Thus, we may write

$$\frac{M_1}{M_2} = \frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\tan \beta_1}{\tan \beta_2} \quad . \quad . \quad . \quad . \quad (3)$$

where the M's denote the moments of the magnets and the α's and β's the corresponding deflections, the subscript 1 refers to one magnet, and the subscript 2 to the other.

Theory.—Let the magnet have poles of strength m situated $2l$ apart, and write $M = 2lm$. Then, assuming the inverse square law, it may be shown that

$$F = \frac{2Md}{(d^2 - l^2)^2} = H \tan \alpha \quad . \quad . \quad . \quad (4)$$

and

$$G = \frac{M}{(d^2 + l^2)^{3/2}} = H \tan \beta \quad . \quad . \quad . \quad (5)$$

so that, if d and l are kept constant the tangent of the angle of deflection is proportional to the moment of the magnet.

Results.—(*a*) From the tabulated values plot curves with $\tan \alpha$ and $\tan \beta$ for one magnet as ordinates, the distances d being the abscissæ.

(*b*) State the ratio of the M's for the two magnets used.

(*c*) Ask a demonstrator the value of H at your work bench, and so obtain an estimate of the actual values of each of the M's.

87. Oscillations of a Magnet.

Apparatus.—Knitting Needles, Bar Magnets, Wood Clip Stand and Suspension Fibres, Two Lead Weights, Paper for Stirrups (or Special Holder).

Objects.—To find how the *period* (or time of vibration) of a magnet depends upon the various circumstances of the

case, and to use that knowledge to compare the moments of two magnets.

Variation of Periods.—Suspend a knitting-needle that is very slightly magnetic and ascertain the period T of its vibrations by timing a counted number, say 10 or 20 complete vibrations. Try to state why the needle vibrates when displaced and let go. Find whether that period is dependent on the amplitude of the swings. Ascertain whether the period is altered if you change the magnetic moment M of the needle by stroking it with the magnet. Try also the effect on the period of changing the field H where the needle vibrates by placing a magnet near it. Test the effect of loading the needle by lead weights, at first near to the suspending fibre, and then as far apart as possible.

In conducting these investigations, observe the important rule of changing only *one* circumstance at a time; since otherwise you could not be sure which change in circumstance was associated with the observed change in behaviour.

Results.—Tabulate all your observations making them as quantitative as you can, and state your conclusions as to dependence of period on (1) amplitude, (2) M, (3) H, (4) loads on the needle, (5) the distance of these loads apart. Find whether your results agree with the relation

$$T^2 \propto \frac{K}{MH} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which K denotes the *moment of inertia* of the needle and its loads about the axis of suspension, and expresses their *sluggishness to start or stop rotating about that axis.*

Comparison of Moments.—Now suspend two bar magnets together with their like poles in the same direction, so that their separate magnetic moments M_1 and M_2 add, and time their swings to find their period, S say. Next place them together with unlike poles in the same direction so that their magnetic moments are opposed, and give an effect represented by their difference and time the period, T say. Then we have by (1)

$$S^2 = \frac{cK}{(M_1 + M_2)H} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$T^2 = \frac{cK}{(M_1 - M_2)H} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where c is a constant. Thus, on division we find

$$\frac{M_1 + M_2}{M_1 - M_2} = \frac{T^2}{S^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

whence

$$\frac{M_1}{M_2} = \frac{T^2 + S^2}{T^2 - S^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

RESULT.—Tabulate your observations and state the conclusion drawn from them as to the ratio of the two magnetic moments. Note that you have compared the magnetic moments of the two magnets *without* determining K or H, which have remained the same during the experiment, and therefore disappear in division.

88. Magnetic Charts.

APPARATUS.—Small Magnetic Needle (suspended or pivoted), Large Sheet of Drawing Paper and Pins; Magnets or Bars of Soft Iron or both, either on, or fixed just under the bench.

OBJECTS.—To explore the field due to any given or specified combinations of magnets or iron bars, seen or hidden, and to make a chart of the field, showing its direction and intensity at a number of points.

METHOD.—It is necessary to find a number of lines of force and the intensities of the field at each of a number of points along each line. To trace the lines, begin at suitably spaced points at a place where the field is weak, tracing each line towards the stronger part of the field. To trace a line, set the needle close to the paper and let it settle (tapping, if pivoted), then mark each end of the needle. Next advance the needle a little more than its own length, and set down again with the rear end near the point previously made to

indicate the fore end, make another mark for the present position of the fore end. Advance again, in like manner, obtaining a new point for each step of the needle through its own length forward. The line drawn smoothly through these points will give the line of force required. The direction of the field should be shown by an arrow on each line.

To ascertain the relative strengths of the field at various points, let the needle oscillate at each place, and count the number N of complete periods in some convenient time, say half a minute. Then N must be inversely proportional to the period T of one oscillation. But T^2 is inversely as the field H in which the needle oscillates (see Experiment 87). Hence

$$N^2 \propto H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

accordingly the square of N serves as a relative measure of H at any point.

Enter near such point the value of N as soon as found, and show also in bolder figures in a ring its square to represent the field strength there, thus: $3^2 = \textbf{9}$, $5^2 = \textbf{25}$; the bolder figures being inside the circle to whose centre the value applies.

Search carefully for any places called *neutral points*, at which the field vanishes.

Results.—Indicate neatly on the chart the lines that best portray the field and show the intensities at a sufficient number of places, making the chart as complete as time allows.

Show also the arrangement of magnets or other bars to which the field was due, and state on it the date, the bench, and the direction of magnetic north.

89. Earth's Magnetic Field.

Apparatus.—Magnetometer, Bar Magnet, Wood Stand, Fibre and Stirrup, Rule, Balance and Weights.

Object.—To determine absolutely the strength of the horizontal component, H, of the earth's magnetic field at the

given place in the laboratory. The magnetic moment of the bar magnet may also be calculated.

Theory.—This experiment consists of two distinct parts. In one, using the bar magnet to *deflect* the needle, the value of the *quotient* $M \div H$ is found. In the other, observing *oscillations* of the bar magnet, the value of the *product* $M \times H$ is found. Hence each of these quantities may be calculated absolutely.

Deflections.—Thus, if in the first (or A) position of the magnetometer, a bar magnet of half length l and of magnetic moment M placed at a distance d, deflects the needle an angle a in a field H due to the earth, we have

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan a = A \text{ say} \quad . \quad . \quad . \quad (1)$$

Oscillations.—Again, if small oscillations of the same bar magnet of moment of inertia K occurring in the field H have period T, we have

$$MH = \frac{4\pi^2 K}{T^2} = B \text{ say} \quad . \quad . \quad . \quad . \quad (2)$$

Moment of Inertia.—The moment of inertia is a purely mechanical quantity depending on the mass and shape of the body, and the axis about which the body is rotating. For a rectangular magnet of mass m, half-length l, and half-breadth b (taken horizontally) oscillating about a vertical axis through the centre, we have

$$K = m\left(\frac{l^2}{3} + \frac{b^2}{3}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a cylindrical body of half-length l and radius c oscillating about a perpendicular central axis, we have

$$K = m\left(\frac{l^2}{3} + \frac{c^2}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Method and Results.—Carry out each part of the experiment carefully in the manner directed in previous experiments on deflections and on oscillations, entering all the

distances, angles, etc., in suitable tables, and showing by sketches the positions used. Calculate by equations (1) and (2) the values of A and B, then we have

$$H = \sqrt{B \div A} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which serves to determine H absolutely as required.

Again, we have $$M = \sqrt{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

so that M should be found also.

90. Susceptibility of Iron.

Apparatus.—Hammer, Long Bar of Iron, Magnetometer Needle (or Compass), Wood Stand, Rule, and Callipers.

Object and Definitions.—If a bar of iron without magnetization is placed in a magnetic field it becomes magnetized, the process being assisted by several smart taps. Let the bar have area a of cross-section, and acquire a pole strength m when placed lengthwise in a field V and tapped.* Then the quotient

$$\frac{m}{a} = I . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is called the *magnetization* of the bar.

Again, the quotient

$$\frac{I}{V} = k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is called the magnetic *susceptibility* of the iron under *tapping*.

It is the object of the experiment to determine k for the specimen of iron provided, the V being the earth's vertical field.

Method and Theory.—To find k we must magnetize the bar in the earth's vertical field and then measure m, which is conveniently done by the deflection α of a needle at a distance CA $= d$, by the pole's field F $= m/d^2$ when perpendicular to the earth's field H. See Fig. 30.

Hence the compass is placed near a corner of the table

* Some prefer to omit the tapping altogether.

and adjusted to zero. Next, the bar is held vertically and tapped smartly several times. Lastly, the bar is placed carefully in a vertical position in the wooden stand, so that one end A is on a level with the compass and in the line passing magnetic east and west through its centre C, the distance CA being ***d***. We then have

$$\frac{m/d^2}{\mathrm{H}} = \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Inserting the value of m from (1) and (2), this may be written

$$\boldsymbol{k} = \frac{d^2}{a} \cdot \frac{\tan \alpha}{\mathrm{V} \div \mathrm{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus it appears that we need only the ratio of V to H, and not the actual value of either. Further, this ratio is the tangent of the angle of dip which was found in a previous experiment.

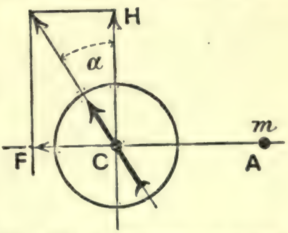

Fig. 30.—Deflection by single pole.

Hence on measuring d, a, observing α, and inserting $\mathrm{V} \div \mathrm{H}$, we may find k.

Results.—The other end B of the bar should also be placed on a level with C, and then the magnetism reversed by placing the bar upside down and tapping again.

Both ends of the needle should be read, all the observations neatly tabulated, and the susceptibility k calculated.

Note.—If the iron is not very soft, it may have permanent poles $\pm m_0$ to which the induced pole m is added. Thus according to which end is down when tapping it, the lower pole may have strength $(m + m_0)$ or $(\boldsymbol{m} - m_0)$. Hence both $\boldsymbol{m}$ and $\boldsymbol{m}_0$ may be found.

PART VI.—ELECTROSTATICS

91. Gold-Leaf Electroscope.

APPARATUS.—Electroscope, Rods of Glass, Ebonite, and Sealing-wax, Pieces of Silk, Fur, and Flannel.

OBJECT.—To familiarize the student with the electroscope and its primary uses.

NOTE.—Consult a demonstrator, if necessary, as to the dryness of the apparatus, since, if moist, leakage will occur and spoil all the exercises.

EXERCISES AND RESULTS.—Carry out the following exercises, showing by sketches and brief descriptions, what you have done and observed in each case.

(*a*) *To charge by direct contact.*—Rub the rod of glass lightly with the silk, cautiously bring the rod near the knob of the electroscope, and, if the divergence of the leaves is not too great, touch the electroscope with the rubbed part of the glass. The leaves are then diverged with positive electricity communicated by the contact. Discharge the electroscope by touching the knob with a finger, when the leaves immediately collapse. Next rub a rod of ebonite or sealing-wax lightly with fur or flannel, again approach carefully, and, *if safe*, touch the knob with the rod. The leaves then diverge through the presence of negative electricity communicated by the contact. Great care is needed in the above cases lest the leaves should be torn. What do you infer from these phenomena as to the attraction or repulsion of like kinds of electricity ?

(*b*) *To charge by induction.*—Rub one of the rods with its appropriate rubber as before, and bring the rod cautiously near the knob of the electroscope till a suitable divergence is

obtained. Then, while holding the rod there with one hand, touch the knob with a finger; the leaves then collapse. Remove the finger, and lastly the electrified rod. The leaves should then diverge again. Why is this, and with what electricity are they now charged?

Carry out the same operations with another rod, giving the other electrification, and note carefully all that happens, stating what you believe as to the electrical state at each stage.

(*c*) *Uncharged bodies on approach reduce the divergence of the leaves.*—To establish this, charge the electroscope and then bring the hand near the knob, when a partial collapse may be seen to occur. Other uncharged bodies (like the metal handle of the glass rod) may be used also. If the collapse is very slight, it may be best detected by holding the body near the knob, and then watching the leaves while the body is *swiftly jerked away.* Try to explain this collapse and re-divergence.

(*d*) *To demonstrate the opposite kinds of electrification.*—Charge the electroscope by induction from the ebonite or sealing-wax, then the leaves diverge with positive electricity. Now rub the glass rod with silk, bring the glass near the knob, and note the result. Also bring near to the knob the sealing-wax or ebonite rod, first uncharged, and then charged by rubbing. Note what happens in each case, and state what you infer. Discharge by touching the knob.

Next charge the electroscope by induction from the rubbed glass rod, so that the leaves diverge with negative electricity. Test their behaviour on bringing near the knob, first a rubbed ebonite rod and then a glass rod before rubbing and after.

(*e*) *To show that friction presents the opposite electricities in equal quantities.*—Wrap a rod of ebonite in its fur rubber, and twist one briskly with respect to the other. Then, without taking the rod out of the fur, bring the two together near to the knob of the electroscope, and try to charge it by induction. Note the result. Next, while both are near the knob, slide the fur off and remove it, leaving the charged rod alone there. What happens now, and why? Have you thus established the theorem?

92. Electrophorus.

Apparatus.—Electrophorus, Fur, Gold-leaf Electroscope, Long Conductor on Insulating Stand, Proof Ball (or Plane), Fine Bare Iron Wire.

Object.—To familiarize the student with the charging and use of the electrophorus.

Description.—The electrophorus consists of an insulating disc (called the *cake*) with tin foil below (called the *sole*) and a loose disc of metal above (called the *cover*).

Exercises.—(*a*) *To charge the electrophorus.*—Strike the upper face of the cake with the fur, and then bring this upper side very *cautiously* towards the knob of the electroscope. When near enough to make the leaves diverge slightly, touch the knob so as to charge the electroscope by induction.

Now place the cover on the cake, and touch the sole and the cover at the same time with a finger and thumb of the same hand. Then lift the cover from the cake, taking care that the coat-sleeve does not come near the cover (for that would discharge it). Bring the cover *slowly* towards the knob of the electroscope, watch the leaves, and note the result.

On the supposition that the cake's upper surface was negatively electrified by striking with the fur, explain the signs of all the other electrifications obtained throughout the process.

Without further striking of the cake by the fur, replace the cover, touch cover and sole simultaneously, lift the cover by its handle and again test its electrification by cautiously approaching the cover to the knob of the electroscope. If it is possible thus to electrify the cover repeatedly without further striking, whence comes the energy of the charge ?

Now, instead of using the electroscope, take a spark from the knob of the charged cover by placing near it a knuckle, or any piece of metal held in the hand. Replace cover on cake, touch cover and sole (or, if the sole is "to earth," touch cover only) and try again to take a spark.

Explain the action as fully as you can.

(*b*) *To charge the long conductor by induction.*—Charge the electroscope positively as at first and then hold the charged cover near one end of the long conductor. While it is there, use the proof ball to test the electrification of the long conductor, first trying the far end by touching it with the ball, and then bringing the ball so charged near the knob of the electroscope. Watch the leaves and note the result. Next test, in like manner with the proof ball, the electrification of the near end of the long conductor, again noting the result. Make a sketch and show the signs of all the charges. If, at either end of the conductor, you fail at first to find the charge that theory predicts, it may be due to a leak of the stand (which needs cleaning or drying) or to hairs bridging over the space between the long conductor and the cover of the electrophorus. Clear these away if present, and try again until successful.

(*c*) *Equal opposite charges are produced by induction.*—After detecting the charges at the ends of the long conductor by induction from the cover, remove the cover and then again test all parts of the long conductor with the proof ball. State clearly what you observe and the conclusions drawn.

(*d*) *Bound charge on earthed conductor.*—Connect the long conductor to *earth* (any water-pipe) by the iron wire, and charge it inductively with the cover as before. Explore it with the proof ball, note the results and explain fully the state of things.

Results.—Make a brief summary of your chief conclusions, showing by sketches what you have done and observed.

93. Distribution of Charges.

Apparatus.—Conductors of various shapes (including Sphere, Long Cylinder, Egg-shape, Hollow Cylinder open at top), Electrophorus, Electroscope, Proof Ball (or Plane), Metal Ball with Hook and Silk Fibre, Fine Bare Iron Wire.

Object.—To find how the charges on the various conductors are distributed over the surfaces.

Exercises and Results.—(*a*) Give each of the conductors in turn a charge from the cover of the electrophorus, testing the charge on the various parts of the surface of each by the proof ball and electroscope. Note specially if any part of the surface has a very great charge, or if any part has no charge at all. Make sketches of the conductors representing the distribution of the charge as though it were something laid on with a variable thickness.

(*b*) Connect the hollow cylinder (or can) by the wire to the knob of the electroscope, taking care that the wire does not touch the table or other articles. Charge the metal ball while hanging by its silk fibre and then lower it cautiously into the can without touching sides or bottom, watching the leaves of the electroscope. Lift the ball out of the can and again lower it, again watching the leaves. Next let the ball touch the inside of the can, observing the leaves carefully while the contact is made. Lift the ball out and lower it with or without contact with the can.

Think carefully as to the state of the electrifications of ball and can at each stage, and make sketches showing your conclusions about them. The exercise with the ball and can is the classic one known as *Faraday's ice-pail experiment.*

94. Condensers.

Apparatus.—Two Small Leyden Jars, Discharging Tongs, Electrical Machine, Slab of Paraffin Wax, Fine Bare Iron Wire (Adjustable Spark Gap).

Object.—To familiarize the student with the charging and discharging of condensers under various circumstances.

Exercises and Results.—(*a*) Place a jar on the bench with its knob touching one terminal of the machine and turn so as to charge the jar. Discharge it cautiously with the tongs and note the spark. *Avoid touching the knob of a charged jar, or the shock may be unpleasant.*

(*b*) With the jar placed as before, add a wire from its outer coat to the other terminal of the machine and again charge. Discharge as before and note the spark again.

What do you conclude about the two charges, and why?

(*c*) Place a jar on the slab of wax and try to charge without any wire to its *outer* coating. Do you succeed, and if not, why not?

(*d*) Arrange the two jars in *parallel* (*i.e.* outer coat joined to outer coat, and knob to knob), the knobs being to one terminal and the outer coats to the other terminal of the machine. Charge the jars and discharge with the tongs carefully, noting the spark obtained. Is it larger than from a single jar?

(*e*) Arrange the two jars in *series* (*i.e.* the knob of one to outer coat of the other), charge and discharge, again noting carefully the spark occurring. How does this spark compare with that from the two in parallel, and with that from a single jar?

(*f*) If an adjustable spark gap is available it may be connected in turn across the terminals of a single jar, the two in parallel and the two in series. Then on turning the machine to charge in each case till a spark occurs at the gap, the comparison of the various charges may be made by counting the turns of the handle in each case. This arrangement accordingly affords a somewhat better test as to the charges in these three cases than the methods of (*a*), (*d*), and (*e*).

95. Potential.

Apparatus.—One Long Conductor and Two Spherical Ones, Proof Ball, Fine Bare Iron Wire, Electroscope, Electrophorus and Fur (Potential Model).*

Object.—To ascertain the potential between the two spheres with and without the long conductor there, one of the spheres being charged and the other *earthed*.

* Sold by J. J. Griffin and Sons, Ltd., Kingsway, London, W.C.

Experimental Arrangements.—Place the three conductors in a line at the same level, the long one between the two spheres and with spaces or gaps of about two inches between each end of the long conductor and a sphere. Then *remove the long conductor*, and about a foot from each of the spheres set the electroscope, fixing on its knob a wire long enough to reach to the spheres. Fasten the other end of this wire to the proof ball so that the metal of the ball is connected to the electroscope. Connect one of the spheres to *earth* by a wire. Charge the other sphere by the electrophorus, and take care that it does not leak away.

Exercises and Results.—(*a*) *Potential between spheres.*—Hold the proof ball near the *earthed* sphere and move it in a straight line cautiously towards the charged sphere, watching the leaves of the electroscope all the time. Now try to draw a curve to represent the values of the potential by its ordinates, the abscissæ representing distances along the line in the space between the spheres.

(*b*) *Potential along Long Conductor and in Gaps.*—Now replace the long conductor and explore with the proof ball as before, beginning at the earthed sphere, moving over the gap to the long conductor, touching and then sliding along it, and, lastly, very cautiously, moving over the gap towards the charged sphere.

Again try to make a curve for potentials in this case, the co-ordinates having the same meanings as in the previous one.

(*c*) Make a sketch of the conductors showing the charges at each part of each (see Experiment 92 *b*), and endeavour to reconcile what you know as to charges with what you have just found as to potential. (The potential model should now be referred to, if available, as it forms a valuable check upon the results obtained, and puts the whole matter very clearly.)

PART VII.—ELECTRIC CURRENTS

96. Cells.

APPARATUS.—Lechanché Cell, Dry Cell, Separate Parts of Dry Cell; Outer Pot, Porous Pot, Copper, Zinc, Dilute Sulphuric Acid (15 volumes of water to 1 of strong acid), and Saturated Solution of Copper Sulphate for Daniell Cell; Voltmeter to 5 volts by tenths (Storage Cell and Arrangements for charging it).

OBJECT.—To familiarize the student with the most important electric cells.

EXERCISES AND RESULTS. (*a*) *Leclanché Cell.*—This cell consists essentially of a zinc and a carbon dipping into a solution of ammonium chloride, the plates being separated by a porous pot. The zinc is amalgamated, and the carbon is surrounded by manganese dioxide and gas-coke powder.

Carefully examine the cell, noting the arrangement of the above parts, and make an explanatory sketch of all you observe, showing a section of the cell.

Connect the voltmeter to the cell and note its E.M.F. (Electro-motive Force) and which plate is the positive (*i.e.* which plate must be connected to the positive terminal of the voltmeter to obtain a reading).

NOTE.—The current passes from the positive plate (carbon) to the negative plate (zinc) outside the cell.

(*b*) *Dry Cell.*—The constituents are like those of the Leclanché, except that some material is used to absorb the liquid and so prevent it from being spilt. The zinc, instead of being a rod, is usually in the form of a cylindrical box to

contain all else. Sketch carefully the separate parts exposed, also find the E.M.F. of the complete cell and note.

(*c*) *Daniell Cell.*—This cell consists essentially of an amalgamated zinc dipping into dilute sulphuric acid, and a copper dipping into saturated solution of copper sulphate, the two liquids being usually separated by a porous pot.

From the parts and chemicals provided build up a Daniell cell, test its E.M.F., note the value and which is the positive plate. Also make an explanatory sketch.

If the zinc needs amalgamating, clean it and carefully apply a little mercury with the brush. When this is properly done there will be no gas given off on putting the zinc into the acid.

NOTE.—The Daniell cell should be *taken apart immediately after use.*

(*d*) *Storage Cell.*—This cell consists essentially of lead plates, one set covered with *brownish* lead dioxide, PbO_2, all dipping in dilute sulphuric acid. Unlike the others mentioned above, it requires to be charged by means of a current from some other source before it is fit to give a current. Note the entire arrangement of the cell and sketch it. Test its E.M.F., and note that the positive terminal is connected with the brownish plates. It is marked red.

If the means are available, charge the cell, asking a demonstrator how to check your connections. Note that the *positive* of the charging battery is connected to the *positive* of storage cell, and also that the E.M.F. of charging battery must be *greater* than that of the cell to be charged. The charging current must be regulated so that it is *not greater* than a certain amount, which is generally indicated on the cell. Sketch the connections, and again test the E.M.F. of the cell after the charging is done.

97. Polarization.

APPARATUS.—Two Leclanché Cells, Resistance Box, Plug Key, Voltmeter to two or more volts by tenths, Morse Key.

OBJECT.—To find how the E.M.F. of a Leclanché cell varies with time when various currents are taken from it.

METHOD.—Connect up the apparatus as shown in Fig. 31, taking care that the *plug key is out* until the right time arrives for inserting it.

Then, by depressing the end, K, of the Morse key, the E.M.F. of the cell, E, may be read on the voltmeter, V. On releasing the key it flies up and makes contact at the other end with the resistance box, R. Hence, if the plug key, P, is inserted, the cell will drive a current through this resistance and polarize itself, for a Leclanché cell is not able to maintain a constant E.M.F. when a current is running. Thus, by adjusting the resistance at will and inserting the plug key, it is easy to ascertain how the E.M.F. varies with time for any current which is being taken from the cell.

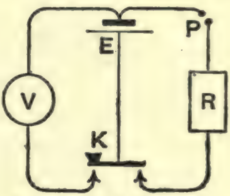

FIG. 31.—Polarization test.

OBSERVATIONS AND RESULTS.—*First*, make the resistance R = 1000 ohms, read the voltmeter before inserting the plug key, and note the result in a column headed "R = 1000." Insert the plug key, and after 1 minute press K and read the voltmeter and enter the result again. Proceed thus, reading every minute up to 15 minutes. The times 0, 1, 2—15 should occupy a column at the left.

Second, make the resistance R = 100, and use the *other cell* while the *first rests and recovers*. Read, insert plug, time, read each minute, and enter in column properly headed as before.

Third, change to the original cell, make R = 10, and proceed as before, entering results in a proper column.

Fourth, change the cell again and make R = 0, reading and entering as before.

Plot four graphs on the same diagram, showing times as abscissæ and volts as ordinates, writing against each graph the value of the corresponding resistance.

What do you conclude about the dependence of polarization (*a*) on time, (*b*) on resistance, and (*c*) on current?

Also, what do you find as to the recovery of the cells after a rest?

NOTE.—If time permits, a Daniell cell may be set up and tested in like manner.

98. Magnetic Field of Current.

APPARATUS.—Box with two Coils in top, Iron Filings, Small Compass, One Storage Cell or Two Daniell Cells, Tapper Key, Soft Iron Bar (about 1 cm. diameter and 10 cm. long).

OBJECT.—To explore the magnetic fields near electric currents in coils, and to make magnets by such fields.

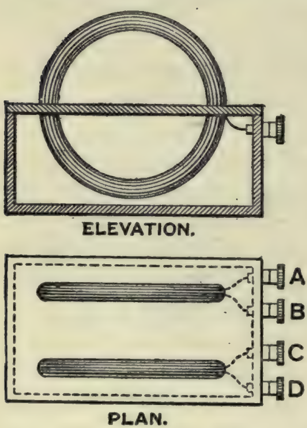

FIG. 32.—Coils for magnetic field.

FIELDS NEAR COILS.—The coil box shown in plan and elevation in Fig. 32 is very convenient for investigating the field of coils. If not available, two coils may be held in clip stands and a card through them supported on a wood block. Sketch the arrangement adopted, trace the connections, and carefully note on your diagram the direction of the wire from each terminal. Since the currents used in this experiment are necessarily large, keep them on for as *short a time* as possible on each occasion.

The general nature of each of the fields can be best explored by the use of iron filings, but the direction must be ascertained by the compass. The battery is connected through the tapper key to the coil or coils in use, the card is tapped,

and the current kept on only a few seconds until the effect is seen. The key is then released and the result entered.

Use (i) either coil alone, (ii) both coils with currents in same direction, and (iii) both coils with opposite currents.

Judging from the fields, in which cases do the coils attract and in which repel one another?

State also the relation which holds between the directions of the current and field, comparing them with the two motions of a screw, and illustrate by a sketch. Motions perpendicular to the plane of the diagram should be indicated by a *circle, with* a *dot* inside if *towards* the spectator, and a *cross* inside if the motion is *from* the spectator.

Find the direction of the current in any wire, from the corresponding field, by means of your observations.

Electro-magnets.—Since a current is accompanied by a magnetic field and a magnetic field magnetizes iron placed in it, we may, by a current, make a piece of iron into a magnet called an *electro-magnet*. Further, when the relation between current and field is known, this knowledge may be applied to produce a magnet with poles of any given size at each end.

Using what you have just established as to directions, wind wire round the iron bar and pass the current so as to produce any prescribed polarity. Thus, make in turn the ends of the bar, N and S, S and N, N and N, and S and S. In each case test each end by the compass, being satisfied that the polarity is properly ascertained only when a repulsion of the needle is observed.

Results.—Describe and sketch all you have done and what you have found and concluded in each case.

99. Ohm's Law.

Apparatus.—One Storage Cell (or two Daniell Cells), Long Wire (about an ohm) on graduated board, Variable Resistance, Ammeter to one ampere by tenths, Voltmeter to two or more volts by tenths.

OBJECT.—To familiarize the student with the relation, called Ohm's Law, which holds between the E.M.F. acting between the ends of any conductor, the *steady current* through it, and the *resistance* of that conductor.

It should be noted that this experiment does *not establish* the law, for it is performed with instruments whose designs and graduations are *based on its assumed truth.*

EXPERIMENTAL ARRANGEMENT.—Connect the ammeter A, the graduated wire GW, and the variable resistance S, in a circuit ready for attachment to the source of current (+ −), then connect the voltmeter to some chosen part R of GW ; all as shown in Fig. 33.

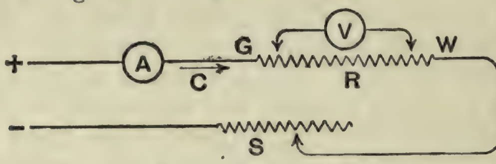

FIG. 33.—Ohm's law.

EXERCISES AND OBSERVATIONS.—*First*, keep the part R unchanged, but vary the resistance S. Read each time the current C by the ammeter and the E.M.F. or potential difference E between the ends of R by the voltmeter. Enter these in separate columns, and in another column enter the values of the quotient $E \div C$.

Second.—Keep the current C constant, but vary the length L of the part R to which the voltmeter is connected. Enter in separate columns the values of L and the readings E of the voltmeter; also in another column enter the values of the quotient $E \div L$.

RESULTS.—Do you find either of the above quotients to be a constant ?

Are all the results in accord with

$$E \div C = R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$R = Lr \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where r is a constant for the given size of wire and its

material? Equation (1) is the symbolic expression of Ohm's law. It is also often written

$$C = \frac{E}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Try to discover and state what error there must be in the assumptions made as to the currents *in the ammeter* and *in the resistance* R. In order that these errors should be small should the resistance of the voltmeter be large or small?

100. Galvanometer Calibration.

APPARATUS.—Dial Galvanometer of either Astatic or Moving-Coil Type, Resistance Box with Infinity Plug, Daniell Cell (or all Parts, etc., for it).

OBJECT.—To find the relation between the current in a galvanometer and the deflection of its needle, since the two are not usually proportional. This process of exploration is called *calibration*.

METHOD.—Connect up in circuit, as shown in Fig. 34, the

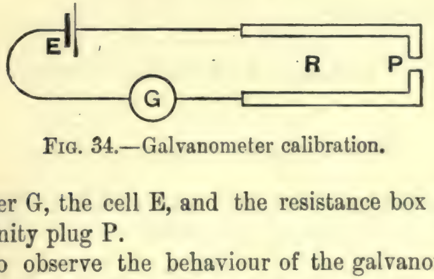

FIG. 34.—Galvanometer calibration.

galvanometer G, the cell E, and the resistance box R, leaving out the infinity plug P.

Then to observe the behaviour of the galvanometer, it is necessary to set it to zero and to use in turn various values of the resistance R, each time inserting the plug P and observing the deflection D° due to the current C. But, on applying Ohm's law to the entire circuit, we have

$$C = \frac{E}{R + G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where G is written for the resistance of the galvanometer (supposed known) and E for the E.M.F. of the Daniell cell, which is 1·07 volt, nearly. The resistance of the cell is here neglected, as it is very small.

To make a satisfactory calibration deflections increasing by about 5° should be observed over the range from 0° to 60° or more, and on *each* side of the zero.

For simplicity's sake, it is well to make each value of $(R + G)$ some exact multiple of hundreds or tens of ohms, letting the corresponding deflections come a little above or below the 5°, 10°, 15°, etc., aimed at, the precise value of such deflection being, of course, noted. In reading each position of the needle both ends should be read, and the eye must be *vertically* over each end in turn *to avoid parallax.*

Results.—Enter all your observations and deductions in tabular form, and then plot two curves having currents as abscissæ and deflections as ordinates, one for the positive currents and one for the negative ones. The currents may be expressed in milliamperes or microamperes (*i.e.* in thousandths or millionths of an ampere). Then the calibration curve makes it possible to use the galvanometer to read the values of any such currents directly.

What do you notice in the curve up to about 20°? What current would be needed for a deflection of 90°?

101. Tangent Galvanometer.

Apparatus.—Tangent Galvanometer, Resistance Box with Infinity Plug, Daniell Cell (or Parts, etc., for it, Centimetre Rule.

Object.—To confirm the tangent law and to determine the reduction factor of the galvanometer while in its present field.

Method.—Connect up as shown in Fig. 35.

Then proceeding as in the previous experiment and tabulating as before the relation between current and deflection is obtained.

Tangent Law.—Further, to test the nature of this relation, tabulate also the quotient $C \div \tan\theta = k$, say. If the quotient is practically constant the tangent law is confirmed.

Reduction Factor and Theory.—The quantity k is called the reduction factor for the magnetic field in which the galvanometer was used, and is thus determined also. But we may with advantage see why it varies with the field and also check its value by the following theory.

From Laplace's law it is known that the magnetic field F at the centre of a short coil of n complete turns of radius r

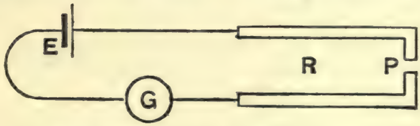

Fig. 35.—Tangent galvanometer.

and carrying a current of C amperes is perpendicular to the plane of the coil and of value

$$F = \frac{C}{10} \cdot \frac{n 2\pi r}{r^2} = \frac{Cn\pi}{5r} \quad . \quad . \quad . \quad . \quad (1)$$

But if, with the coil in the magnetic meridian of a field H, a current gives a deflection θ to a very short needle at its centre, then

$$F = H \tan\theta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, equating the right-hand sides of (1) and (2) we have

$$C = \frac{5r}{n\pi} H \tan\theta = k \tan\theta \quad . \quad . \quad . \quad (3)$$

It is thus seen that k is proportional to H, and its dependence on the number and radius of the windings of the galvanometer is also shown. Accordingly, by counting and measuring the turns in the coil, it can be found what value of H is needed to fit that of the k, already determined. Ask a demonstrator

if your value is correct. Remember the value of H in the laboratory is not necessarily that in iron-free space in the vicinity.

RESULTS.—Show clearly, with sketches and tables, all you have done. State also the deflections in the present field for a tenth and a half ampere. What would these become if the field H were halved or trebled?

102. Galvanometer Shunts.

APPARATUS.—Galvanometer and Shunts, Resistance Box, Bar Magnet and Clip Stand, Daniell Cell.

OBJECT.—To show that shunts fail to control the galvanometer's deflections unless there is appreciable resistance in the main circuit, and to explain this.

EXPERIMENTAL ARRANGEMENT.—Connect up the apparatus as shown in Fig. 36, in which E denotes the cell (and its E.M.F.), I and C the currents in the main circuit and galvanometer respectively, R, G, and S the resistances in the main circuit (including that of the cell), the galvanometer and the shunt respectively. The shunts are supposed of the ordinary type, such that

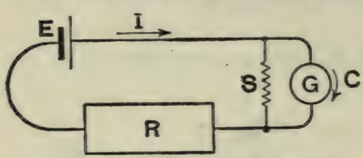

FIG. 36.—Galvanometer shunts.

$$S = \frac{G}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $n = 10$, 100 or 1000 at will. Thus, when the shunt is not in use $S =$ infinity and $n =$ unity.

METHOD AND EXERCISES.—*First:* Make R as small as possible, *i.e.* put no resistance in with the box, so that R reduces to that of the cell only. Inquire what this value is and note it. Then, using no shunt, place the magnet in the clip stand to control the galvanometer to obtain, with the current on, a suitable deflection; say the length of the scale, if a mirror

galvanometer; or 40° if an astatic dial galvanometer. Note this deflection for no shunt under the heading $n = 1$. Next, leaving the magnet as set, use the shunt so that $n = 10$, and again note the deflection. Do the same with the other shunts for which $n = 100$ and 1000. This completes the first set of readings for the given value of R.

Second: Put in such resistance with the box as to make (with the battery resistance), R = one-thousandth of G. Set the control magnet so that with no shunt the current gives the same deflection (or nearly the same) as in starting the first set of readings. Then, leaving the magnet untouched, proceed to try all the shunts in order as before, noting the deflections each time.

Third: Make R = one-hundredth of G, and read and note as before.

Fourth: Make R = one-tenth of G.

Fifth: Make R = G.

Sixth: Make R as large as possible, each time carrying out the readings with the various shunts.

Tabulate all your results as neatly as possible. Plot curves, one for each value of R, all on one diagram, in which deflections are ordinates, and the abscissæ are 0, 1, 2, and 3 for *no* shunt, $n = 10$, 100, and 1000 respectively. (It will be seen that the abscissæ are the values of $\log n$.)

THEORY.—By applying Ohm's law we may establish the following results:—

$$\text{Resistance of galvanometer and shunt} = \frac{GS}{G+S} = \frac{G}{n} \quad (2)$$

$$\text{Main current} = I = \frac{E}{R + G/n} \quad . \quad . \quad . \quad (3)$$

$$\text{Fraction of it through galvanometer} = \frac{C}{I} = \frac{S}{G+S} = \frac{1}{n} \quad (4)$$

$$\text{Then (3) in (4) gives } C = \frac{E}{G + nR} \quad . \quad (5)$$

This last very important expression for the galvanometer current, explains what has been experimentally found, viz.—

(1) That, while R is so small that nR is inappreciable in comparison with G, the shunt's power (expressed by n) is practically absent, the galvanometer current remaining nearly E/G, whether the shunt is in or not.

(2) That, if R is so large that nR is distinctly appreciable in comparison with G, then, and then only, the shunt acts as intended and sensibly reduces C.

Results.—Show all observations neatly tabulated, and describe any difficulties met with in the experiment. Do your observations and curves all agree with equation (5) ?

State clearly in words how the shunts may fail to reduce the galvanometer current, although they only pass part of the main current through it.

103. Copper Voltameter.

Apparatus.—Copper Voltameter with three plates, Storage Cell (or Daniell's), Adjustable Resistance, Plug Key, Ammeter to one ampere by tenths (or Tangent Galvanometer).

Object.—To measure a current by the mass of copper deposited in a given time, and to use this measure to test an ammeter (or galvanometer).

Ampere by Voltameter.—The voltameter method of measuring current has proved to be so good that it has been adopted for one of the definitions of the practical unit. Thus, subject to certain conditions, one *ampere* may be taken as that current which deposits *copper at the rate of* 0·000329 *gm. per second* from a solution of pure copper sulphate in distilled water (the density of the solution being 1·18 gm. per c.c., and 1 per cent. of sulphuric acid then added to it).

Experimental Arrangement.—The apparatus is joined up in simple circuit as shown in Fig. 37, in which LL denote the outer bars from which the two *loss* plates hang into the solution of copper sulphate, and G the middle bar from which the *gain* plate or *kathode* hangs. This plate must be joined to the negative terminal of the cell E, so that the current *leaves* the voltameter by G. This ensures that G shall *receive* the

deposit, and on *both* sides since it is between the other two plates LL, which are the *anode* of the voltameter.

A is an ammeter, of which the readings are to be tested, R is a variable resistance, and P is a connecting plug.

METHOD.—Take care that the voltameter plates are clean, washing, if necessary, in dilute nitric acid, also clean the contacts with sandpaper. Place the three plates in position in the voltameter. Calculate what current is admissible, allowing not more than one-fiftieth of an ampere per square cm. of surface of copper plate deposited on, then adjust the variable resistance so as to pass the desired current. Let the current run for about five minutes to obtain a good surface on the gain plate. Stop the current, remove the gain plate, wash it in tap water, finishing with water containing a trace of sulphuric acid, then dry by waving the plate quickly some distance above a bunsen flame. As soon as cold, weigh the plate correctly to a milligram.

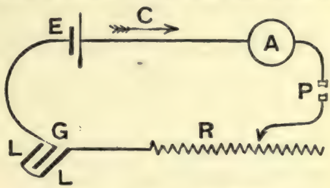

FIG. 37.—Copper voltameter.

Insert in the voltameter, and allow the current to run for a timed period of 20 to 30 minutes, watching the ammeter the whole time and adjusting the resistance, if necessary, so as to keep the current constant at the desired value.

At the end of the period, stop the current, remove the gain plate, wash, dry, cool, and weigh as before.

Let the weights of the plate before and after the deposit be W_1 and W_2 gms. respectively, the period be *t minutes*, and the true value of the current be C amperes. Then, from the definition given before, we have

$$C = \frac{W_2 - W_1}{60t \times 0{\cdot}000329}$$

So, if the ammeter read C′, its error would be the difference $C' - C$.

(If a tangent galvanometer is used, its deflections right and left should be tested, and its reduction factor found.)

If time permits, the whole process should be carried out for some other *smaller* current; or for a *larger* current if *two* voltameters are available to be placed in *parallel*, *i.e.* the loss plates of both connected together to one wire, and the gain plates of both to another.

RESULTS.—Write an account of your arrangements, operations, observations, and conclusions.

104. E.M.F.'s of Cells.

APPARATUS.—Daniell Cell, Leclanché Cell, some other low-resistance Cell, Resistance Box with infinity plug, Galvanometer.

OBJECTS.—To compare the E.M.F.'s of the ordinary cells by three simple methods and, assuming that of the Daniell to be 1·07 volt, to find in volts the values for the other cells.

EXPERIMENTAL ARRANGEMENT.—Connect up in series as shown in Fig. 38, one cell, the galvanometer and the resistance box with the infinity plug out.

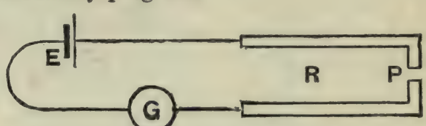

FIG. 38.—E.M.F.'s of cells.

CONSTANT CURRENT METHOD.—Set the galvanometer to zero and adjust the resistance to obtain a suitable deflection (say 40°), and note it together with the value R_1 of the resistance and the name of the cell whose E.M.F. is E_1, say.

Replace this cell by another whose E.M.F. is E_2, say, and find the value R_2 of the resistance needed to give again the *same* deflection as before. Then, *if the resistance of the cells are negligible* it follows from Ohm's law that

$$\frac{E_2}{E_1} = \frac{R_2 + G}{R_1 + G} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where G denotes the resistance of the galvanometer. A different deflection may now be used, and each cell tried again.

The third cell may be compared with the first in like manner. If it were found desirable to shunt the galvanometer with a shunt of resistance

$$S = \frac{G}{n - 1}$$

then the G in (1) would be replaced by $(G \div n)$.

Again, if G or $(G \div n)$ were negligible in comparison with R_1 and R_2, equation (1) would reduce to

$$\frac{E_2}{E_1} = \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

In any case R_1, R_2 should be large so as to prevent the cells from polarizing and to enable one to neglect the resistances of the cells.

Constant Resistance Method.—In this method a resistance is found which gives suitable deflections with each cell in turn. Then, the ratio of currents C_1 and C_2 is found (1) by the tangents of the angles if a tangent galvanometer is used, or (2) by the known calibration curve if an astatic galvanometer is used. Thus, the resistance being practically constant in the two cases, we have

$$\frac{E_2}{E_1} = \frac{C_2}{C_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here again R must be large to prevent cells from polarizing.

In most dial galvanometers if the deflections are kept small, say not more than 30°, the deflections are nearly proportional to the currents. This is also almost exactly true when a mirror galvanometer is used. Thus, if the deflections are kept small, the deflections are proportional to the currents, so that the E.M.F. may be compared, even though the calibration curve of the galvanometer is not known.

Sum and Difference Method.—In this method, as its name implies, we use first the *sum* and then the *difference* of

the E.M.F.'s to drive the current, and through the *same* resistance.

Thus, with the cells E_1 and E_2 both in the circuit and tending to send the current the *same* way round, suppose the current is S amperes. Next with the cells opposing each other, suppose the current D amperes flows and in the direction of E_2 the second cell.

Then by Ohm's law, we have

$$\frac{E_2 + E_1}{E_2 - E_1} = \frac{S}{D},$$

whence

$$\frac{E_2}{E_1} = \frac{S + D}{S - D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is easily seen that the expression on the right-hand side of equation (3) is calculable if only the ratio of S to D is known. And this ratio of currents is found from the deflections in the manner already described.

Here again the resistance in the circuit should be large.

Results.—Give all your observations and deductions neatly tabulated, with diagram and descriptions of the methods used.

Using the Daniell's cell as the standard, calculate the E.M.F.'s of the other cells.

105. Potentiometer for E.M.F.'s.

Apparatus.—Galvanometer, Simple Potentiometer, two Daniell Cells (or one storage), as source of current, one Daniell, one Leclanché, and one other cell for comparison.

Experimental Arrangement.—The form of potentiometer for this experiment is simply a long fine uniform wire AB mounted on a board graduated to a hundred or more divisions. Its ends are connected to the source of current S as shown in Fig. 39, the resulting current being C amperes. The cell E under

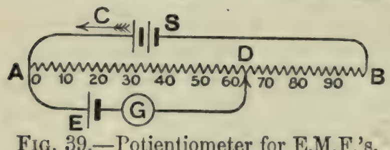

Fig. 39.—Potientiometer for E.M.F.'s.

examination, and the galvanometer G, are put in a branch circuit from one end A of the potentiometer to a variable contact point D. It should be noted that the point A must be joined to plates *of the same sign* in the cell under examination and in the main battery or source of current S. Also the E.M.F. of the source S must exceed that of any of the cells to be compared.

METHOD.—Let the E.M.F. of the cell under test be E_1 volts, and find a point D_1 on the wire such that *no current* then passes through the galvanometer.

Then, obviously, the points A and D_1 are at the potential difference E_1. Further, let AD_1 include d_1 divisions of the wire and its resistance be r ohms per division. We accordingly find by Ohm's law that

$$E_1 = Crd_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

NOTE.—The student should *not* be satisfied with any point D_1 as the balance point *unless near points at opposite sides give opposite deflections.*

Next replace the cell by a second one whose E.M.F. is E_2, and find for it a balance point D_2 at a distance of d_2 divisions from A. Then, as before, we have

$$E_2 = Crd_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus, by division

$$\frac{E_2}{E_1} = \frac{d_2}{d_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence, if one of the cells is a Daniell of known E.M.F. = 1·07 volt, that of the other may be calculated in volts.

Repeat for the third cell and find its E.M.F.

ADVANTAGES OF METHOD.—The potentiometer method is a very good one, because no current passes through the test cell, and therefore neither its resistance nor its polarization comes into account.

RESULTS.—Give a diagram of the method used, tables of all observations made and results deduced.

106. Thermoelectric Cell.

APPARATUS.—Galvanometer (preferably with a low-resistance *moving coil* and mirror), thick Iron Wire, Clip Stand, Daniell Cell, Resistance Box, Tripod, Wood Block, two Beakers, Thermometer to 100° C.

OBJECT.—To explore the behaviour of a thermoelectric cell of iron and copper, qualitatively to a red heat and quantitatively to 100° C.

QUALITATIVE EXERCISE.—To the ends of a piece of the iron wire twist two copper wires and connect them to the galvanometer, holding one junction C of iron and copper in the clip stand so as to expose the other to the hottest part of a bunsen flame; all as shown in Fig. 40.

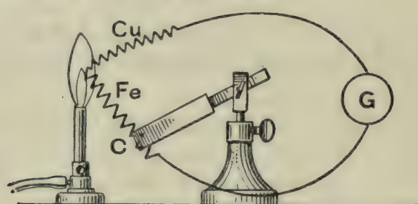

FIG. 40.—Thermoelectric cell.

Watch and record the deflections of the galvanometer as one junction is heated slowly to a bright red heat and then allowed to cool slowly, the other junction being all the time cool.

Now replace the iron wire and bunsen by the resistance box and Daniell cell, and thus determine roughly the order of magnitude of the greatest E.M.F. obtained by the thermo cell as well as its sign, *i.e.* whether from copper to iron, or iron to copper, at the hot junction. (Inquire from a demonstrator the resistance of the galvanometer in use.)

QUANTITATIVE EXERCISE.—Set a thermo cell in the circuit again with the iron wire just long enough to bring the junctions into separate beakers of water, as shown in Fig. 41.

The junction C is now kept cold in a beaker of water on the wood block, while the junction H is heated in the beaker on the tripod over the bunsen flame, the thermometer being clipped in position with its bulb level with H.

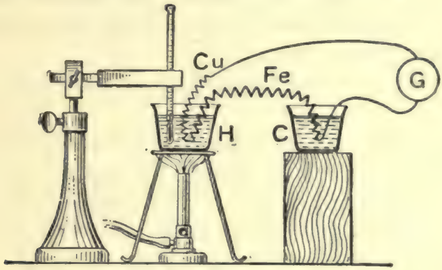

FIG. 41.—Thermoelectric E.M.F.

Heat up gradually, and read temperatures and deflections simultaneously at about every 10° up to 100° C.

RESULTS.—Sketch the arrangements adopted, tabulate your observations, and plot curves for them, showing temperatures as abscissæ and E.M.F.'s as ordinates.

Also state how you could measure temperatures electrically.

107. Resistance by Substitution.

APPARATUS.—Two or more Resistances to measure, Resistance Box, Galvanometer and Shunts, Daniell Cell.

METHOD.—Connect one of the resistances in series with the cell and the galvanometer. (If necessary, refer to a demonstrator as to the method of using the shunts.) Then arrange the shunts so as to obtain a suitable deflection (30° or 40° say), and note it carefully.

Disconnect the resistance under test, and put the resistance box in its place. Vary the resistance in the box till you obtain exactly the same deflection as before, *the shunts remaining unaltered.* Then the resistance previously used must be equal

to that of the known resistance now in the box, for the substitution of one for the other is the only change made.

Find the smallest variation of resistance in the box which causes a perceptible variation of the deflection, and hence calculate the percentage accuracy of your measurement.

Repeat the operation with each resistance provided, thus obtaining the value of each.

Also measure the resistance of all (1) *in series*, and (2) *in parallel.*

RESULTS.—Sketch the arrangement used, tabulate all your observations, and check by theory the values found for the resistance in series and in parallel. Remember that, in *series*, the total resistance is the *sum of the resistances;* but that in *parallel*, the *sum of the reciprocals* of the resistances is equal to the *reciprocal* of that resistance which might replace the set.

108. Resistance by Slide Bridge.

APPARATUS.—Slide Wire Bridge, Galvanometer, Leclanché Cell, One Coil of known resistance (or a Resistance Box), Two or more Coils whose resistances are to be measured.

THEORY.—Suppose that four resistances P, Q, R, S are joined up end to end in a quadrilateral form ABCD as shown in the diagram of Fig. 42.

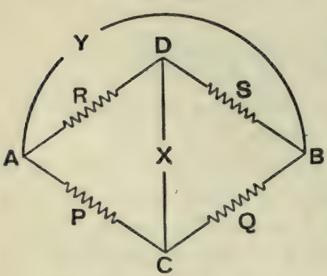

FIG. 42.—Bridge diagram.

And in either of the gaps X and Y in the two *diagonals* let a battery be introduced, a galvanometer and key being then placed in the other gap.

Suppose, further, that on pressing the key, no current passes through the galvanometer; then an important relation between the resistances can be established as follows.

Imagine the galvanometer to be at X, then the points CD

are at the same potential, since no current flows between these points. Also the current I, say, through P continues unchanged through Q; and the current J, say, through R continues unchanged through S. Then since the fall of potential from A to C must equal that from A to D (which is at the same potential as C), we have

$$IP = JR.$$

Similarly, for the parts, CB and DB, we have

$$IQ = JS.$$

Then, by cross multiplication, we obtain

$$PS = QR \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, in other words, *the product of one pair of opposite resistances equals the product of the other pair.*

The symmetry of this result shows that if we placed the galvanometer at Y, and the battery at X, the same relation would hold. But the student is advised to establish this independently if he feels any doubt on the matter.

If we regard S as the unknown resistance to be measured, we may transpose (1) into the form

$$S = \frac{Q}{P} R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This shows that, of the other three resistances, it suffices to *know one and the ratio* of the other two.

These four resistances are often termed the *arms* of the bridge.

Experimental Arrangement.—The resistances P and Q are now to be formed by the two parts into which the slide wire AB is divided by the contact point C. The known resistance R is placed in one of the back gaps of the bridge, and the coil S to be measured in the other gap, all as shown in Fig. 43. Further, the galvanometer is placed in the gap called X in the previous figure, and the Leclanché cell in Y.

Method.—Thus the test is made by finding the point C,

which gives no current through the galvanometer. Near points should also be found which give opposite deflections, so that it is known to what accuracy the balance point is determined. Then, since the slide wire is supposed to be of uniform diameter and material, the resistances P and Q may be taken as proportional to the lengths AC and CB as read on the scale. Hence by equation (2) the resistance S is calculated.

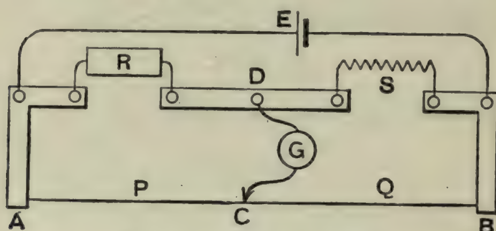

FIG. 43.—Slide wire bridge.

As a check, R and S may be interchanged and C found again. If R is adjustable, it is better to use it with such resistance as to bring C near the centre of AB.

EXERCISES AND RESULTS.—Measure the resistances of all the coils provided, singly, in series, and in parallel. Also check these last results by theory. Show by diagrams and brief statements all you have done and found, and state the percentage accuracy of your measurements.

109. Resistance by Box Bridge.

APPARATUS.—Box Bridge, Galvanometer, Leclanché Cell, Two or more Coils whose resistances are to be measured.

EXPERIMENTAL ARRANGEMENTS.—In the box form of the bridge three resistances are provided, as shown by P, Q, and R in Fig. 44. The unknown resistance S is accordingly connected to the points B and D, thus completing the *four arms* needed to form the bridge. The galvanometer key, and a key for the battery are also usually provided

in the box, as shown by K_2 and K_1. It is customary to place the battery between C and D through K_1, and the galvanometer between A and B through K_2, see Fig. 44. It may be noted that this arrangement illustrates the interchange of battery and galvanometer previously referred to as possible. (See Experiment 108 on the Slide-wire Bridge).

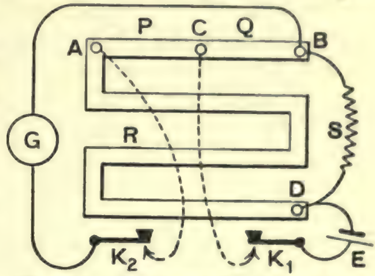

FIG. 44.—Resistance by box bridge.

RANGE AND SUBDIVISION.—P and Q are called the *ratio arms*, and each can usually be made at will 10, 100, or 1000 ohms. R is called the adjustable arm, and can usually be increased by single ohms from 0 to 10,000.

But, recalling the relation between the arms when the balance is reached, we have

$$S = \frac{Q}{P} . R.$$

Thus, inserting on the right the possible values for such a box,

S may be measured *up to* 1,000,000 ohms,
or *down to* 0·01 ohm.

Further, in this range we may proceed

By steps of	0·01 ohm up to		100 ohms
„	0·1	„	1000 „
„	1	„	10,000 „
„	10	„	100,000 „
„	100	„	1,000,000 „

Of course all this is on the supposition that the galvanometer is sensitive enough to respond to each change in the box.

NOTE.—Where *various* values of the ratio arms will give the ratio required, it is better to use values near those of the

R and S. For it may be shown that the bridge is in the most sensitive state when all four resistances are equal.

METHOD.—Having made the arrangement shown in the figure, and taken out plugs in each of the arms P, Q, and R, the key K_1 is put down and held while the key K_2 is put down to test if any deflection occurs. If the galvanometer gives no indication, it is not right to assume off-hand that the balance-point has been found. It may be that some connection is faulty. The student should *never be content with no deflection* for one adjustment until he has found *opposite deflections* for near adjustments on each side of the adjustment giving no deflection. Further, these opposite deflections and the changes in adjustment producing them should be noted to ascertain to what accuracy you are able to measure. If the galvanometer is not sensitive, or the student feels a doubt as to the best use of shunts, if any, a demonstrator should be consulted.

EXERCISES.—Measure the resistances of the coils provided, singly, in series and in parallel, and check the latter results by theory.

RESULTS.—Give a careful diagram of your arrangement, and tabulate all observations and results, showing also the accuracy of the latter.

110. Specific Resistance.

APPARATUS.—Box Bridge (or Slide Bridge), Galvanometer, Leclanché Cell, Set of Coils of Wires of various known lengths, diameters, and materials (or, Loose Wires of various sizes and materials, Metre Rule, Micrometer Gauge).

OBJECT.—(1) To derive a formula giving the resistance of a wire in terms of its length, cross-section, and material.

(2) To evaluate in this formula the constant characteristic of the material, and called its *specific resistance.*

METHOD AND EXERCISES.—Beginning with wires of a given diameter and material, find by the bridge test how the

resistance varies with length, and embody the result in a preliminary formula. Next, keeping length and material constant, vary the diameter, or vary the cross-sectional area by placing two, three, or more wires side by side and connected in parallel. Note how resistance varies with cross-section, and combine this second result with the first into one formula, testing carefully that it expresses both experimental laws. Put this formula in the form of an equation with the resistance on one side, then it will have a constant on the other. Expressing all your dimensions in centimetres and resistance in ohms, the constant (corresponding to the resistance when the length and cross-section are each unity) is the *specific resistance*, and is expressed in cm. ohms.

Having found this for one material, proceed to test the formula and evaluate the constant again for a second material, and so for all the metals or alloys provided.

RESULTS.—Explain carefully your mode of experimenting and reasoning. Calculate also for some of the wires the length which would have a resistance of one ohm, and check this experimentally.

Do all your observations agree with the equation

$$R = r\frac{l}{a},$$

where R is resistance in ohms of a wire of length l cm., cross-sectional area a sq. cms., and specific resistance r cm. ohms?

111. Low Resistance.

APPARATUS.—Standard Low Resistance of about a hundredth of an ohm, Thick Wires of Copper or other Metals of about a thousandth of an ohm resistance to measure, Sensitive Galvanometer with Mirror, Storage Cell (or Two Daniell Cells), Tapper Key.

EXPERIMENTAL ARRANGEMENT.—The bridge method is not suitable for these very low resistances as the ordinary

contacts and leads would present a greater resistance than that under test. The following *direct comparison of potentials* is perhaps the simplest method to use first. The standard resistance R is placed in series with that to be measured X, the tapper key K and the storage or Daniell cell E. If two Daniell cells are used they would probably give more current if placed in parallel, as shown. Then the galvanometer G is connected in turn (1) to A and B, the ends of R, and (2) to C and D, the ends of X. See Fig. 45.

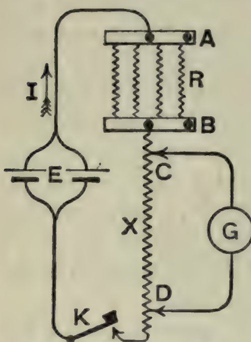

FIG. 45.—Low resistance.

THEORY.—Let the current in the main circuit be I, then the potential differences at the ends of the two resistances will be IR and IX. Dividing these by G, the galvanometer resistance, we obtain the corresponding currents in the galvanometer which will be proportional to the deflections r and x, say.

We may accordingly write

$$\frac{IR}{G} = kr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\frac{IX}{G} = kx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is some constant depending on the sensitiveness of the galvanometer, whence, on dividing, we obtain

$$\frac{X}{R} = \frac{x}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Of course G must be large in comparison with R and X in order that no appreciable part of I may branch off into the galvanometer circuit, and thus make the I different in equations (1) and (2).

EXERCISES.—Test all the wires or rods provided, and check

by putting them in series. State fully what you have done and found.

Why would it be incorrect to use a Leclanché cell for this experiment, and yet be correct to use one in the bridge test?

112. High Resistance.

APPARATUS.—Resistance Box to 10,000 ohms, Sensitive Mirror Galvanometer with Shunts, Pot Insulators and can of water with Brad-awl for contact (or other Resistances up to 1000 megohms), Several Lelanché Cells.

EXPERIMENTAL ARRANGEMENTS AND THEORY.—*First*, join up in simple circuit the insulation resistance X, the galvanometer G and the battery E; the contact being made with the brad-awl B on the upturned spike of the pot in the water, all as shown in Fig. 46.

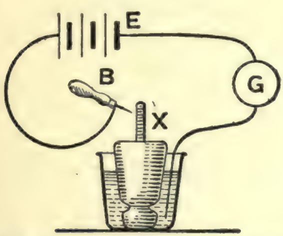

FIG. 46.—Insulation resistance.

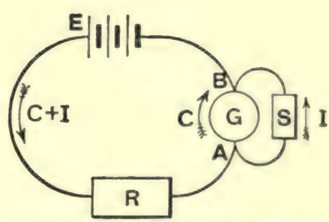

FIG. 47.—Galvanometer test.

Then, if the galvanometer deflection is x scale divisions, we may write for the current the two equal expressions

$$\frac{E}{X} = kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant depending on the galvanometer. It is here assumed that the resistance of the galvanometer is negligible in comparison with X.

Second, replace the insulation resistance by the resistance box R and use the shunt $S = G \div (n-1)$ say, as shown in Fig. 47.

Let the current C pass through the galvanometer and the current I through the shunt S. Then, for the potential difference between A and B, we have the three equal expressions

$$CG = IS = \frac{IG}{n-1}$$

whence $$C(n-1) = I, \text{ or } C = \frac{I+C}{n}$$

where I + C is evidently the main current $E \div R$, say, provided the resistance of the shunted galvanometer is negligible in comparison with R.

Thus, if the galvanometer now gives a deflection of r scale divisions, we may write

$$C = \frac{E}{Rn} = kr \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

provided the sensibility of the galvanometer is unchanged.

Hence, dividing (2) by (1) we eliminate E and k, and find

$$X = \frac{nrR}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Accordingly, if $R = 10{,}000$ ohms, $n = 1000$, $r = 100x$, then $X = 1{,}000{,}000{,}000$ ohms or 1000 megohms.

Exercises and Results.—Using the arrangements described, test the resistances provided, tabulating neatly your observations and conclusions.

Note.—The wire from the battery to the brad-awl should *not touch anything else.*

Inquire the value of the galvanometer resistance G, and show that the resistance of the shunted galvanometer was negligible in comparison with R, and that G itself was negligible in comparison with X, as assumed in equations (1) and (2).

113. Galvanometer Resistance.

APPARATUS.—Galvanometer with Control Magnet and Shunts, Box Bridge, Leclanché Cell.

EXPERIMENTAL ARRANGEMENT.—The galvanometer to be tested is joined up in the usual place for the unknown resistance in the box bridge as shown in Fig. 48. The battery also occupies its usual place. Then, instead of a galvanometer in the path A to B through K_2, we put a wire simply. We have thus no direct means of detecting any current in this branch. But we detect it, if any, by the *change* in the current through S, which is always flowing when the battery is on. To observe this change properly; after putting K_1 down, the galvanometer deflection should be brought back to about zero by the control magnet, then K_2 is put down and the change, if any, noted.

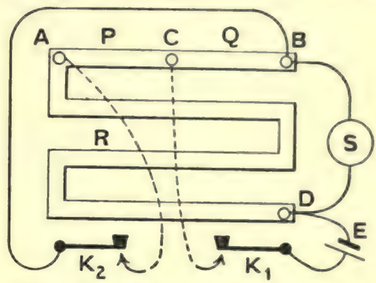

FIG. 48.—Galvanometer resistance.

Thus, no adjustment of P, Q, and R can be expected to prevent the main deflection which occurs first. The object being simply to prevent any *change* of that deflection occurring on putting down K_2. For if no change occurs, it is clear that no current has passed between A and B.

EXERCISES AND RESULTS.—Measure the resistance of the galvanometer shunted in its possible ways and unshunted, describing carefully your methods throughout. Also check the relations between the various values found for the resistances.

114. Battery Resistance.

APPARATUS.—Resistance Box with Infinity Plug, Voltmeter to two or more volts by tenths, Tapper Key, Various Cells to test.

Experimental Arrangement.—Connect the voltmeter V and tapper key K at one side of a cell (or a battery of several cells), also connect at the other side the resistance box R with infinity plug P out, all as shown in Fig. 49.

Method and Theory.—*First:* With plug P out press

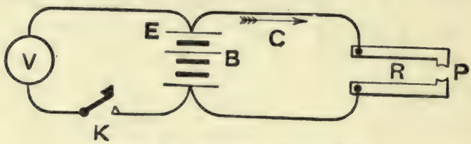

Fig. 49.—Battery resistance.

key K, and obtain on voltmeter the reading E volts which is the E.M.F. of the battery.

Second: With plug P in, but a certain resistance R in use in the box, press key K and obtain on voltmeter the reading F volts.

Then this is the potential difference or voltage at the terminals of the bat ery and at the terminals of the resistance R. Thus, if the current through the resistance box is C, we may write

$$F = CR.$$

But, denoting by B the internal resistance of the battery, we see that

$$C = \frac{E}{B + R}$$

Hence $$F = \frac{ER}{B + R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which B is the only unknown, so may be calculated.

The same expression for F is found, if we begin by regard ing it as

$$F = E - CB.$$

Graphic Illustration.—Both views are very instructively illustrated if we make a diagram in which resistances are measured horizontally and voltages in a perpendicular direction as shown in Fig. 50.

In this the E.M.F. is E, and the total resistance B + R; hence the current is given by the slope of the hypotenuse, or

$$C = \frac{E}{B + R} = \tan A = \frac{F}{R}$$

whence

$$\frac{E}{F} = \frac{B + R}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From either (1) or (2), or Fig. 50, it is seen that, if F = half E, then B = R.

PRECAUTIONS.—(1) The current should only be allowed to run just long enough to allow of reading the voltmeter, lest polarization should occur, and the E.M.F. of the battery should therefore not be the same when the two voltmeter readings are taken.

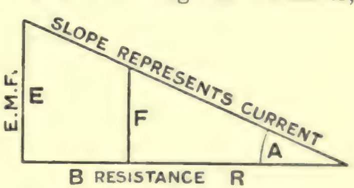

FIG. 50.—Voltage and resistance diagram.

(2) The resistances used in the box must not be too small, for the same reason.

(3) It is desirable to take several values of R, beginning with fairly large ones and approaching the case where F is half E.

(4) If the internal resistance of a cell is low, it is better to use three of them in series, set two against one, as shown in Fig. 49. This increases the internal resistance without increasing the E.M.F., and allows you to increase the external resistance to match, and so improves the conditions of the experiment.

EXERCISES AND RESULTS.—Observing the above precautions, test the various cells provided and neatly tabulate your observations and results, giving also a statement of the theory involved.

115. Electrolyte Resistance.

Apparatus.—Slide Wire Bridge, Resistance Box, Telephone Receiver, Cell and Small Induction Coil without Condenser, Copper Sulphate Solution and Copper Electrodes in Glass Tube, Steel Rule.

Experimental Arrangement.—Connect the cells E to the primary of the induction coil I, and the secondary of the coil to the ends of the bridge as shown in Fig. 51. Connect

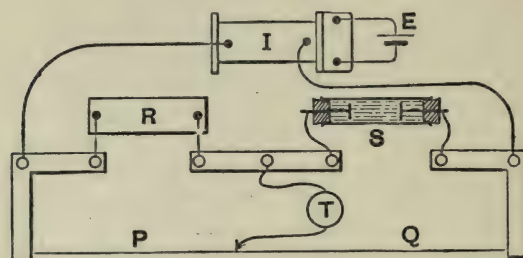

Fig. 51.—Electrolyte resistance.

the box R and the unknown electrolyte resistance S in their usual places, and complete with the telephone T instead of the galvanometer.

Thus, in this arrangement, *alternating* currents are derived from the coil and detected by the telephone instead of the continuous ones used in the ordinary test for metal resistance. The alternating currents are here adopted to minimize the polarization of the electrolyte, and the consequent error involved by the use of continuous currents.

Method.—Start the induction coil and find the balance point as that at which the sound either vanishes or is a minimum. Then the usual bridge relation holds, viz.—

$$PS = QR$$

whence S the resistance sought.

If the sound does not quite vanish anywhere, locate the point of minimum sound as that midway between those each side which give equally feeble sounds. This determination should be carried out several times and a mean taken.

Further, to secure the most sensitive arrangement, the box should be adjusted to such a resistance as will bring the balance point near the centre of the wire.

Exercises and Results.—Find the resistances of columns of various lengths of the electrolyte. Measure the internal diameter of the tube and so calculate the specific resistance of the sample of copper sulphate solution under test.

Show clearly by diagrams and descriptions all you have done and found.

State what is meant by the polarization of the electrolyte mentioned above. Why should it affect the determination of the resistance by continuous current?

116. Temperature Variation of Resistance.

Apparatus.—Slide Wire Bridge, Galvanometer, Leclanché Cell, Two Coils of about equal resistance, one being of Fine Iron Wire with Stout connecting pieces of copper (a Resistance Box may be used for the other), Beaker, Tripod, Thermometer to 100° C., Wood Clip Stand.

Method.—Connect up for the bridge test, the coil of iron wire being arranged immersed in water in the beaker mounted on the tripod. Set the thermometer in place held by the clip stand so that its bulb is in the water on a level with the coil. Find the resistance of the coil in the cold water and note its temperature. Next, warm the water 20° or 30° by a bunsen, remove the flame, stir the water well. Then find again the resistance and note the corresponding temperature. Repeat the warmings through 20° or 30°, stirrings, and resistance tests till the temperature is nearly 100° C. If time permits, take several tests while the temperature is successively lowered by replacing some of the hot water by cold, each time stirring well.

Results.—Explain by a sketch your experimental arrangement. Tabulate your observations and plot curves co-ordinating resistance and temperature.

Find from this graph the increase of resistance per ohm of original resistance per degree rise of temperature.

117. Test of Lamp for Resistance and Power.

Apparatus.—Glow Lamp to run at four (or six) volts, Two (or Three) Storage Cells, Suitable Voltmeter and Ammeter. (Various lamps and appropriate Cells are desirable.)

Experimental Arrangement.—Connect up the battery E in series with the lamp L and the ammeter A. Then put the voltmeter in shunt so as to include both lamp and ammeter, *i.e.* at points F and H on Fig. 52, and *not* at points G and H simply.

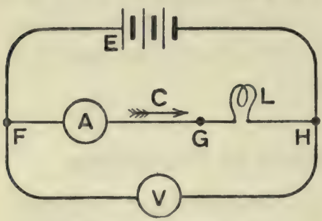

Fig. 52.—Lamp test.

Exercises and Results.—Read the volts and the amperes when the current is running, and note both. From the known resistance of the ammeter (ask its value if not marked on) find the potential difference between F and G, and calculate that between G and H. Having thus the potential difference between the terminals of the lamp itself and the current passing through it, find by division its resistance in ohms, and by multiplication the *power* in watts. Find the number of ergs of energy dissipated in the lamp per minute. (If other lamps are available repeat the test and calculation for each.) Sketch your arrangement and give all your observations and results.

118. J by Current.

Apparatus.—Heating Coil (of about one ohm resistance) in Beaker on Cotton Wool, Thermometer to 40° C. by tenths, Clip Stand, Variable Resistance, Plug Key, Ammeter to two

amperes by tenths, Voltmeter to two volts by tenths, Two Storage Cells, Balance and Weights.

OBJECT.—To find how the heat produced when a current passes through a resistance depends upon the various circumstances of the case and so to obtain *an estimate of* **J**, *the mechanical equivalent of heat.*

EXPERIMENTAL ARRANGEMENT.—Connect up, as in Fig. 53, the battery E_1, plug key P, ammeter A, heating coil R, and variable resistance S, all being in series. The voltmeter is in

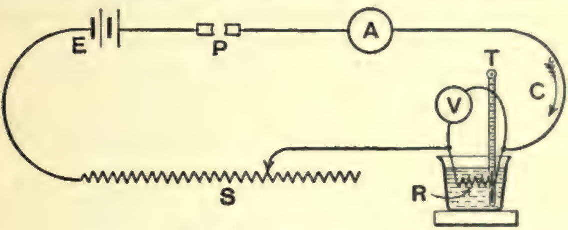

FIG. 53.—J by current.

shunt on the heating coil, which is in water in the beaker on cotton wool, with the thermometer T held in the clip stand, so that its bulb is on a level with the coil.

DEPENDENCE OF HEAT ON VOLTAGE AND CURRENT.—If w gms. of water are raised $r°$ C., the heat H expended is given in calories by

$$H = wr \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But of the heat supplied by the coil to the water in the beaker some will escape by conduction and radiation. To correct for this, having found by trial the adjustment of S to obtain a suitable current, stop the current and watch the fall of the temperature for, say, five minutes. Then run the current for double that time—ten minutes, again watching the temperature change, also the current C and voltage V. Finally, stop the current and watch the cooling for the same period as at first—five minutes, say. Call these three changes of temperature a, b, and c respectively, writing them all

positively. Then it may be seen that the true rise r, if no heat escaped, would be the simple sum of the three changes noted.

Or $$r = a + b + c \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus on substitution of this equation in (1), the heat is expressed in observed quantities.

Tabulate carefully all your observations and repeat the experiment with different values of V, C, and t, the time in seconds which the current was allowed to run.

Also add columns for the product $\mathrm{VC}t$ and for the quotient $\mathrm{VC}t \div \mathrm{H}$.

ESTIMATE OF **J**.—If the quotient just named has a constant value, it is clearly desirable to have a single name and a symbol for it. It is called *the mechanical equivalent of heat* and is denoted by **J**.

If V is in volts, C in amperes, t in seconds, and H in calories, **J** will be in joules per calorie. It is well to find also the reciprocal of **J**, because a convenient way to express the heating effect of a current is

$$\mathrm{H} = \frac{1}{\mathbf{J}}\mathrm{VC}t \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If we knew the exact resistance R of the heating coil at the temperatures of its use, we could dispense with the voltmeter, because then V would be CR.

This shows that we might write instead of (3) the other very common form—

$$\mathrm{H} = \frac{1}{\mathbf{J}}\mathrm{C}^2\mathrm{R}t \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

RESULTS.—Explain carefully with sketches all you have done, show all observations neatly in tables, and give the values you find for **J** and for its reciprocal, stating the units in each case.

Q

119. Comparison of Capacities.

Apparatus.—Standard and other Condensers, Sensitive Mirror Galvanometer, with freely-swinging Coil (or Needle), Several Leclanché Cells, Morse Key.

Experimental Arrangement.—Connect one of the condensers S to the battery E, the galvanometer G, and the Morse key K, as shown in Fig. 54. Hence on depressing K the condenser is charged; and on releasing K the condenser is discharged *through the galvanometer*. The condenser may then be replaced by another and the test repeated.

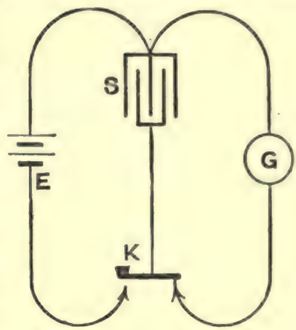

Fig. 54.—Comparison of capacities.

Theory.—The capacity of a condenser in the quotient charge divided by difference of potential of its plates. Or

$$S = Q \div E.$$

Thus the quantity required to charge it is given by

$$Q = SE \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence the quantities charging different condensers to the same potential difference will be proportional to their capacities and serve as a measure of them.

Again, it can be shown that the *first swing* (*or throw*) of a freely-swinging galvanometer coil (or needle) is nearly proportional to the total quantity of electricity Q that is discharged through it when in the zero position.

Or $$Q = kD, \text{ nearly} \quad . \quad . \quad . \quad . \quad (2)$$

where D is the number of scale divisions of throw, and k is a constant depending on the galvanometer.

Thus $$Q = SE = kD.$$

And if for a different capacity S′ we had the throw D′, we might write

$$S'E = kD'.$$

Whence, on division, $$\frac{S'}{S} = \frac{D'}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Or, in words, the capacities of the two condensers are directly proportional to the first throws of the galvanometer.

Exercises and Results.—Compare the condensers provided, with the standard, also with each other, and note if the results agree. Give a diagram of your connections, and state how, with the same apparatus, you could compare the E.M.F.'s of batteries.

120. Induction of Currents.

Apparatus.—Primary and Secondary Coils, Iron Wire Core and Bar Magnet to fit the Coils, Storage Cell, Tapper Key, Sensitive Mirror Galvanometer with freely swinging Coil (or Magnet), Compass.

Object.—To find how currents circulate in a coil in whose vicinity magnets are moved or currents changed.

Induction by Magnet.—Connect the galvanometer to the secondary coil (which has *many turns of fine* wire). Place the magnet near this coil and along its axis, and then move one relatively to the other. If the galvanometer has a moving magnetic needle, this motion should be obtained by moving the coil, the bar magnet being kept stationary. But if the galvanometer has a moving coil, the bar magnet may be moved towards the secondary coil ; as this motion will not directly affect the galvanometer apart from the secondary coil. The student should test this point for himself. Also, if a moving coil galvanometer is in use, note that you can quickly stop its swings by short circuiting it. Indeed, the coil here acts like the motors of some electric trams when the emergency

brake is applied. This is an interesting example of Lenz's law, the motion of the coil induces a current of such sign as to stop that motion. These preliminaries being grasped, examine systematically and note the currents that occur with the following relative motions of bar magnet and coil.

(*a*) North pole of magnet thrust into the coil.

(*b*) North pole drawn back again.

(*c*) North pole taken on and out (after being thrust in).

(*d*) South pole of magnet used instead of North as in (*a*), (*b*), and (*c*).

(*e*) Try effect of motions, *quick* and *slow*, each through the *same* range.

Does any current occur owing to the *position* of the magnet or only owing to some *change* in that position? Does the current last *after* the relative motion ceases, so far as you can judge?

Try to find whether all your observations accord with the following relations, in which e denotes mean induced E.M.F. in the secondary coil when the number of magnetic lines through it change from N_1 to N_2 in time t by cutting the coil, R denotes the total resistance of the secondary coil and galvanometer, C the mean current and Q the total quantity passing through the galvanometer.

$$e = \frac{N_1 - N_2}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$C = \frac{e}{R} = \frac{N_1 - N_2}{Rt} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$Q = Ct = \frac{N_1 - N_2}{R} \quad . \quad . \quad . \quad . \quad (3)$$

Remember that the first swing of a *ballistic* galvanometer is proportional to Q.

If the coil in use and the galvanometer permit of it, note the directions of windings of each and so find the actual directions of the currents induced and ascertain if Lenz's law is confirmed.

INDUCTION BY CURRENTS.—Leaving the secondary coil

connected to the galvanometer, connect the cell and tapper key to the primary coil (which has *few turns of thick* wire). Place the compass near one end of the primary coil, press the tapper key, and so find the polarity of the primary coil when the current is on. Next place the primary coil a little away away from the secondary, their axes being coincident, as shown in Fig. 55.

Try to predict what will happen in the galvanometer when any change of position or current is made in the primary coil used instead of the magnet in the former part of the experiment. Then test your predictions and note the results.

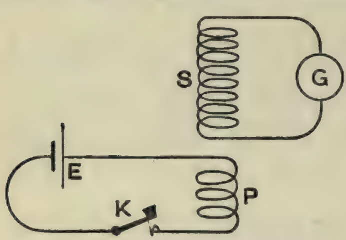

FIG. 55.—Induction by currents.

Remember that the approach of a magnet may be imitated by an approach of the primary coil with the current on, or by putting the current on while the coil is near. Further, the currents may be varied and specially the effects may be changed by the introduction of an iron core wholly or partially into the primary coil. Make every test you can think of. Find all directions of currents (if possible), and state whether Lenz's law is confirmed.

RESULTS.—Explain fully, with diagrams and tables, all you have done, observed, and concluded.

NOTE ON INDUCTION COIL.—The second part of the experiment illustrates the principle of the *induction coil.* In this important apparatus the e of equation (1) is made very great by special arrangements which make the difference of the N's as large as possible and the t as small as possible. An induction coil should now be examined if possible and its action noted. If the parts of one are available it is still better to put them up in order and set the coil going to give as large a spark as can be wisely obtained from it.

TABLES OF LOGARITHMS AND OF TRIGONOMETRICAL RATIOS

N 2

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
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46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
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55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
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57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
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62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
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66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
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70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
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79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
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83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
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88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
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92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
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95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Angle.									
Degrees.	Radians.	Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees
								Angle.	

INDEX

The References are to the Pages.

THE END

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